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Focus Report

Framework for analysing cost and risk for central government debt

A simulation analysis

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Preface

The Swedish National Debt Office manages the central government finances and has a key role in the Swedish economy. The agency's responsibilities include central government cash management, government borrowing and debt management, providing state guarantees and loans, and securing the financing of nuclear waste management. Under its mandate to safeguard financial stability, the Debt Office collaborates with the Ministry of Finance, the Riksbank, and the Swedish Financial Supervisory Authority. The Debt Office is, among other things, responsible for the crisis management of banks and ensuring that there are well-functioning deposit insurance and investor protection schemes.

The Debt Office's Focus Reports explore, and present analysis on, various topics within the agency's areas of operation. These reports serve to illuminate key topics on which we provide in-depth expertise for both the Debt Office's regular target groups and a wider audience. The series also provides the agency's employees with the opportunity to publish analyses externally and thereby obtain valuable input from outside the organisation.

With this series, we aspire to increase awareness and understanding of the focus of our operations as well as to contribute to further discussion. Discourse on the Debt Office's topics is important to us as an agency and in the broader socioeconomic context.

Karolina Ekholm
Director General of the Debt Office

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Summary

This Focus Report presents a framework for analysing how the composition and term to maturity of the central government debt affect its cost and risk. We use a model to simulate future interest rates and inflation starting at the end of 2024. On the basis of the simulations, we calculate cost and risk for different debt portfolios.

In the next stage, we identify an efficient frontier consisting of the portfolios with the lowest cost given a certain risk level. We also create hypothetical portfolios that reflect the Debt Office's current strategy. Comparing the hypothetical portfolios with one another and with the efficient frontier is an important starting point for determining the balance between cost and risk. The central government debt composition that is considered most cost-effective may vary over time on the basis of new data and conditions.

The findings, based on existing data, confirm the established conclusion that portfolios with longer maturities involve higher expected cost but also lower risk. The analysis also shows that inflation-linked bonds have both higher cost and risk than nominal bonds.

The results also show that the hypothetical portfolios similar to those under the Debt Office's current strategy have a higher cost than the efficient frontier for the same level of risk. The additional cost is most distinct for the hypothetical portfolio with a greater proportion of inflation-linked bonds. We conclude that the differences in both expected cost and risk between the hypothetical portfolios are small.

The framework contributes to the Debt Office's overall assessment of the central government debt's cost and risk. The results and conclusions should be weighed together with other analyses and expertise.

Introduction

The aim of this report is to present a framework for analysing the central government debt's cost and risk by comparing debt portfolios that have different compositions and terms to maturity. The framework supports the Debt Office's work to minimise the cost of the central government debt over the long term while taking account of the risk in its management.

For our calculations, we refer to the Government's current guidelines for debt management and use cost measured as the average issue yield on the basis of the valuation principle of amortised cost with continual revaluation of inflation and exchange rate fluctuations. Risk is defined as cost variation for the above measure.

Choosing the central government debt's composition and term to maturity involves a trade-off between cost and risk. We expect to pay a higher cost for borrowing in longer maturities when term premia are positive. At the same time, longer maturities entail lower risk because fixed-income securities with long maturities have less frequent turnover. Long rates have often been less volatile than short rates, which also helps lower the risk for a debt portfolio with a long term to maturity.

Cost and risk are also affected by the central government debt's composition. The choice between nominal bonds and inflation-linked bonds is an important aspect of the composition. Nominal bonds provide investors with a fixed coupon rate (nominal yield) regardless of the outcome of inflation. For inflation-linked bonds, investors are instead compensated for the realised inflation and the Debt Office thereby bears the inflation risk. Whether or not inflation-linked bonds provide a cost advantage over nominal bonds depends on how much investors collectively value protection against inflation.

In the first stage of the framework, we use a dynamic Nelson-Siegel model (DNS model) to simulate interest rates and inflation ten years forward from the end of 2024. With the aid of these simulations, we then obtain expected cost and risk for different terms to maturity and compositions.

We create hypothetical portfolios that reflect the Debt Office's current strategy. These are compared with an efficient frontier consisting of the portfolios with the lowest cost given a certain risk level. A comparison between these hypothetical portfolios as well as a comparison in relation to the efficient frontier provide insights into how the Debt Office can make trade-offs between expected cost and risk.

According to the World Bank, scenario-based models are used by several different countries, particularly to aid in balancing cost and risk.[1] For our model, we have

[1] See The World Bank (2017) *Government Debt Management: Designing Debt Management Strategies*.

drawn inspiration mainly from Italy and Canada.[2] The model does not capture all aspects of debt management and does not replace the Debt Office's other experiences and assessments. However, together with other conclusions, the model's results may be used to support the decision-making process.

The Debt Office has previously analysed the central government debt's cost and risk in the annual proposed guidelines. The framework we present in this report provides a more detailed analysis of costs and cost variation in relation to previous analyses. The analysis methods we employ here are intended to complement the other methods, not to replace them.

In the next chapter, we present the method and discuss conditions. This is followed by descriptive statistics for interest rates, cost and risk, as well as the analysis where we show how the framework can be used. From then on, we compare alternative methods and examine how the choice of time period affects the results. At the end of the report, we present our conclusions.

[2] See Bernaschi et al. (2019), Bolder & Deeley (2011), as well as Audet et al. (2025).

Framework for analysis of cost and risk

This chapter describes the framework, which in simplified terms consists of three parts. The first part covers the DNS model that is used to model and simulate interest rates – zero-coupon yields – and inflation. (In this report, we use the terms interest rate and zero-coupon yield interchangeably, as well as abbreviated variations thereof such as rate and yield, unless otherwise specified.) In the second part, we use the simulations to obtain measurements of cost and risk for different debt portfolios. The portfolios are composed of nominal bonds with different terms to maturity, and inflation-linked bonds. Finally, we calculate an efficient frontier of debt portfolios with the lowest cost given a certain risk level.

Simulation of interest rates and inflation

In order to simulate future interest rates and inflation, we use a variant of the dynamic Nelson-Siegel model (DNS model), which employs a two-step estimation procedure.[3] In the first step, we model the yield curve according to the Nelson-Siegel model (NS model). The model describes the zero-coupon yield $r_t(\tau)$ at the time t with maturity τ with the aid of four parameters:

$$r_t(\tau) = \beta_{0,t} + \beta_{1,t} * \left(\frac{1-e^{-\frac{\tau}{\lambda}}}{\frac{\tau}{\lambda}}\right) + \beta_{2,t} * \left(\frac{1-e^{-\frac{\tau}{\lambda}}}{\frac{\tau}{\lambda}} - e^{-\frac{\tau}{\lambda}}\right).$$

Equation 1

The first three parameters in Equation 1 are time-dependent and can be interpreted as the yield's long-term level $\beta_{0,t}$, slope $\beta_{1,t}$, and curvature $\beta_{2,t}$. The fourth parameter λ is assumed to be constant over time and determines how quickly the function for $\beta_{1,t}$ approaches zero and where the function for $\beta_{2,t}$ reaches its maximum. The first three β-parameters are time-dependent and used as state variables in the time-series model presented below.

In the second step, we model the state variables' development over time with the aid of a vector autoregressive (VAR) model. The model is specified as follows:

[3] Our DNS model uses a two-step estimation procedure in the same way as in Diebold & Li (2006), but it also includes macro variables and parameters for real interest rates. Our model is thus similar to Audet et al. (2025), Bernaschi et al. (2019), and Holler et al. (2018) who use a two-step model with both macro variables and several interest rate parameters. The model can also use a single-step estimation procedure with a state-space structure, as well as be made arbitrage-free and then include terms that adjust the interest rate in a way that ensures discounted bond prices are semi-martingales. The two-step model we use is, however, simple and numerically stable and provides according to Diebold & Rudebusch (2013) estimates equivalent to those using more advanced alternatives. For a more thorough presentation of different DNS models, we refer to Diebold & Rudebusch (2013).

$$Y_t = \mu + \phi Y_{t-1} + \varepsilon_t.$$

Equation 2

Where Y_t is a vector of monthly time series for eight state variables: rate of inflation (inf_t), growth of industrial production (ipg_t), three parameters for nominal yields ($\beta_{0,t}^N, \beta_{1,t}^N, \beta_{2,t}^N$), and three parameters for real yields ($\beta_{0,t}^R, \beta_{1,t}^R, \beta_{2,t}^R$). The rate of inflation is calculated on the basis of the consumer price index (CPI) as annual percentage change. The VAR (1) model allows the state variables to affect one another with a one-month lag.

With the aid of the estimated μ and ϕ as well as a series for the stochastic error term (ε_t), we can simulate the state variables' future values, e.g. values at t+1, t+2, etc. Y_{t+1} is expressed, for example, as follows:

$$Y_{t+1} = \hat{\mu} + \hat{\phi} Y_t + \varepsilon_{t+1}.$$

Equation 3

ε_{t+1} is a random selection with replacement of the historical error terms according to Equation 2. The reason for this choice, instead of assuming a normal distribution, is that this method better captures uncommon periods of high volatility, such as the global financial crisis of 2008 and Covid-19 pandemic in 2020.

The simulated state variables are used further for calculating zero-coupon yields according to the NS model Equation 1.

Measures of cost and risk for funding strategies with individual instruments

According to the Government's current guidelines, the cost of the central government debt is to be calculated using the valuation principle of amortised cost taking accrued inflation and exchange rate fluctuations into account. We follow this principle and calculate the cost on a monthly basis for every funding strategy and simulation.[4] These monthly costs are added together to an annual frequency that forms the basis for our evaluation of cost and risk.

To analyse the portfolios with different compositions, we start with the portfolios that follow the funding strategies in individual instruments. These strategies involve the debt being funded over time with the same type of instrument, both in regard to maturity and type (nominal or inflation-linked bonds). One example of this is a portfolio that is built by regularly issuing only ten-year nominal bonds. Borrowing for these strategies occurs once a month and each strategy's nominal amount over time amounts to one krona. The monthly refinancing requirement is a reciprocal value of the chosen fixed maturity expressed in months. A funding strategy with ten-year nominal bonds involves for instance borrowing 1/120 krona

[4] An alternative concept is cash-based yields, which entail costs being booked when the actual cash flows occur.

every month, whereas a funding strategy with five-year nominal bonds involves borrowing 1/60 krona every month.

Cost measure

With the aid of zero-coupon yields and inflation, we calculate costs for coupon payments and inflation compensation for the central government debt that remain constant. The costs are therefore based on par yield defined as the coupon rate whereby the bond's price, i.e. amount on issuance, is equal to its nominal amount. Since the borrowing is conducted every month at par yield, there are no premia or discounts. At the end of every month, there is an equal amount maturing as there is being refinanced (e.g. 1/120 krona if the borrowing is in ten-year bonds). The costs are thereby consistent with the valuation principle of amortised cost but are simpler to calculate.

The input for the calculations is zero-coupon yields, $y_{i.t}$, with maturity *t* from 1 to n years and *i* is the bond's type (1 for nominal and 2 for inflation-linked). The zero-coupon yield on a government bond is the annual return an investor can expect from investing in a zero-coupon bond over its term to maturity according to the following formula:

$$y_{i,t} = \sqrt[t]{\frac{Nominal\ amount}{Price}} - 1.$$

Equation 4

The Debt Office does not issue zero-coupon government bonds, but the zero-coupon yields can be calculated from market interest rates and market prices for government bonds.[5]

The coupon payment, or par yield, $c_{i,n}$, for a bond with maturity *n* is then provided by the following expression if both the issuance amount and nominal amount for the bond are set to one krona:[6]

$$\text{Issuance amount} = \frac{c_{i,n}}{(1+\mathrm{y}_{i,1})} + \frac{c_{i,n}}{(1+\mathrm{y}_{i,1})^2} + \cdots + \frac{c_{i,n}}{\left(1+\mathrm{y}_{i,n}\right)^{\mathrm{n}}} + \frac{\text{Nominal amount}}{\left(1+\mathrm{y}_{i,n}\right)^{n}},$$

which can be written as

$$c_{i,n} = \frac{1-\frac{1}{(1+y_{i,n})^n}}{\sum_{t=1}^{n}\frac{1}{(1+y_{i,t})^t}}.$$

Equation 5

This formula is applied for each month for both nominal and real rates with a maturity from one to ten years. Except for keeping the debt constant, par yield also

[5] For a more detailed description of how zero-coupon rates for government bonds can be calculated, see for example Hull (2005), page 82. Hull calls these *treasury zero rates*.

[6] To calculate the cost of funding strategies with individual instruments, par yield is multiplied by the issuance amount, for instance 1/120 krona for a funding strategy with ten-year nominal bonds or 1/60 krona for a funding strategy with five-year nominal bonds.

takes into account that the coupon payments are made annually (see the calculation example in the footnote).[7]

The annual cost for par yield, $c_{i,n,}$ for nominal bonds is then distributed over the months, i.e. a twelfth for each month. For inflation-linked bonds, continual revaluation for inflation is applied. Both the coupon and the nominal amount are indexed to CPI. The total cost for an inflation-linked bond (i=2) during its term to maturity of *n* years and nominal amount one krona is as follows:

$$K_{i,n} = \underbrace{\sum\nolimits_{t=1}^{n} \frac{I_t}{I_b} * c_{i,n}}_{\text{Coupon cost}} + \underbrace{\frac{I_n - I_b}{I_b}}_{\text{Inflation compensation for nominal amount}} .$$

Equation 6

I_t is CPI-lagged by three months at year t. I_b is the bond's base index that is measured at the time of issue. The first term indexes the coupon cost and the second term captures the change in inflation compensation on nominal amounts. We calculate monthly costs for inflation-linked bonds by following the same principle with continual revaluation for inflation. The appendix contains a detailed description.

For every funding strategy and simulated inflation and interest rate path *s*, we calculate an average annual cost, $\overline{K_{i,n}^{s}}$, over the evaluation horizon that is set at ten years. Finally, we calculate the average cost over all simulations ($S = 10{,}000$) for each funding strategy as follows:

$$\overline{K_{i,n}} = \frac{\sum_{s=1}^{S} \overline{K_{i,n}^{s}}}{S}.$$

Equation 7

Risk measure

To capture how much more a funding strategy in certain conditions costs compared with its average cost, we use the risk measure of relative Expected Shortfall (rES) at a 95 per cent confidence level.

$$rES_{i,n} = \frac{\sum_{\overline{K_{i,n}^{s}} \geq CaR_{i,n}} \overline{K_{i,n}^{s}}}{(1-\alpha) * S} - \overline{K_{i,n}}$$

Equation 8

[7] The following calculation example describes par yield for a nominal bond with a maturity of two years, $c_{1,2}$, where the one-year zero-coupon rate, $y_{1,1}$, is 1 per cent and the two-year zero-coupon rate, $y_{1,2}$, is 4 per cent. $c_{1,2} = \frac{1 - \frac{1}{(1+4\%)^2}}{\frac{1}{(1+1\%)^1} + \frac{1}{(1+4\%)^2}} = 3.9\%$. This bond pays an annual coupon of 3.9 per cent, and the borrowing amount on issuance is the same as the nominal amount on maturity.

The first term measures the average cost of the simulations that are above a high threshold value ($CaR_{i,n}$ with a confidence level α at 95 per cent). The other term measures the average cost of all simulations. A high rES value indicates that the investment strategy shows high cost variation, which indicates a higher level of risk.

Two types of debt portfolios

We construct debt portfolios by combining funding strategies with individual instruments that each consist solely of a bond of a certain type and term to maturity. There are a total of 20 funding strategies that consist of nominal and inflation-linked bonds with maturities between one and ten years. The cost of a debt portfolio, K_p, is thereby a weighted sum of costs for the funding strategies. The weight of every individual funding strategy ($w_{i,n}$) is to be positive and is added up to one for each portfolio.

$$K_p = \sum_{i=1}^{I=2} \sum_{n=1}^{N=10} K_{i,n} * w_{i,n}.$$

Equation 9

With the aid of the cost of a portfolio that is described in Equation 9, we apply Equation 7 and Equation 8 to calculate the average cost, $\overline{K_p}$, and risk, rES_p, for the portfolio.

We create two types of debt portfolios by combining, in two different ways, the individual-instrument strategies. The first type, hypothetical portfolios, reflect the Debt Office's current strategy, and Table 2 shows their compositions. We construct, for instance, a portfolio with a large share of inflation-linked debt and compare it with another with a lower share. The other hypothetical portfolios vary in term to maturity. All the debt portfolios are unchanged over time, e.g. the share of inflation-linked debt is ten per cent for portfolios 1-5 throughout the time period.

The other type of debt portfolio, the efficient frontier, combines funding strategies with individual instruments through optimisation. By solving an optimisation problem, we identify a debt portfolio with the lowest cost for a certain risk level. We can, in other words, create an efficient frontier consisting of debt portfolios with the lowest cost for their risk level. The efficient frontier is a theoretical exercise expressed as:

$$\arg\min \left\{ \overline{K_p} \middle| rES_p \leq risk\ level, \sum_{\mathrm{i}=1}^{\mathrm{I}=2} \sum_{\mathrm{n}=1}^{\mathrm{N}=10} \mathrm{w}_{i,n} = 1 \,, \mathrm{w}_{i,n} \geq 0 \right\}$$

Equation 10

The above equation presents the efficient frontier that consists of ten portfolios that have the lowest cost for a given level of risk. Each portfolio is in turn a combination of funding strategies with individual debt instruments that are nominal and inflation-linked bonds with maturities between one and ten years. In total, there are 20 funding strategies with individual instruments. The weight of every funding strategy ($w_{i,n}$) is to be positive and is added up to one for a portfolio for which the average cost, $\overline{K_p}$, is the lowest for the risk level.

The weight for each funding strategy is constant over time. In the future, we may develop the analysis of the debt portfolios in different ways. On such addition would be to allow the need for funding to vary over time. We can also seek an efficient frontier for which we let the weights vary over time (i.e. the proportion of inflation-linked bonds in the portfolio increasing or decreasing with time).[8]

Framework assumptions are important to consider

When interpreting the results, it is important to consider the underlying assumptions. One of these is that the Debt Office's actions do not affect the pricing of the bonds in each maturity. This means, for example, that the yield on a one-year bond will not change regardless of the supply. Nevertheless, in reality we cannot change the volume too much without affecting the pricing situation.[9] The purpose of our analysis is not to estimate what the pricing would have looked like if the Debt Office were to have issued in another manner. The analysis focuses instead on hypothetical portfolios that closely resemble the historical issuance patterns.

Another important assumption is that all funding strategies involve issuing at a constant amount over time and the borrowing being conducted at par yield. With this assumption, the funding amount is unchanged over time, which simplifies the estimation of costs. Interest costs and inflation compensation affect the cost but not the size of the debt. In reality, the Debt Office's borrowing is not conducted at a completely even pace and the cost of the borrowing may affect the size of the debt. This assumption helps to, in a stylised manner, present cost and risk for different debt portfolios.

The simulations in this analysis are based on the historical relationship between the chosen state variables. The relationship is estimated with the help of the VAR model. Data for the model covers the period from October 1997 to December 2024. The time horizon is relatively long, but it is not certain that forthcoming trends for market interest rates and inflation will follow historical patterns. Accordingly, the results may be interpreted differently as new data comes in.

Another assumption is in regard to the evaluation horizon. We consider the evaluation horizon of ten years to be sufficiently long for capturing the progression in interest rates between their level at the start of the evaluation horizon and the model's long-term equilibrium values (mean values observed in historical data). Short-term interest rates may, for example, be low from the start and rise towards the long-term equilibrium values, but they may also be high then drop in periods

[8] Audet et al. (2025) analyse, for instance, dynamic strategies in which a portfolio is re-weighted depending on how market interest rates change.

[9] In-depth discussions on the impact of the Debt Office's actions are found in the reporting of the findings from the Government assignment to examine whether the evaluation of the overall debt-management objective can be made easier (Reg. no. 2016/1345).

ahead, which we see in the 2025–2034 period. A long evaluation horizon of ten years is also essential for comparing long-term costs for the funding strategies with different terms to maturity in this study. The longer the funding strategy's term to maturity is, the more time it takes for the entire debt to be refixed at potential new interest rates.

A further assumption is the choice of cost measure. The Debt Office follows the Government's current guidelines and measures cost as the average issue yield on the basis of the valuation principle of amortised cost with continual revaluation of inflation and exchange rate fluctuations. We conclude that the Debt Office has historically used different cost definitions and that the choice of cost measure may affect the findings in this report. The fact that inflation-linked bonds are continually revalued in relation to inflation outcomes affects how much risk is considered to be associated with these instruments.

The framework also relies on assumptions of a technical nature. One of these is how we estimate interest rates with the NS model. The interest rates according to the state variables generated by the NS model deviate from the observed market rates, and the difference varies over time. Another assumption is that we simulate interest rates and inflation with what is known as the bootstrap method. This means that a random selection of error terms is used as the stochastic component for every time step in the simulation. If future trends in, for example, inflation are not as volatile as historical trends, our bootstrap simulations capture a more extreme stochastic component than if we had used normally distributed error terms.

Cost and risk for funding strategies with individual instruments

In this chapter we present stylised facts for funding strategies with individual instruments that each consist solely of bonds with the same term to maturity and type (either nominal or inflation-linked bonds). First, we present the yields for nominal bonds and inflation-linked bonds respectively and then the costs.

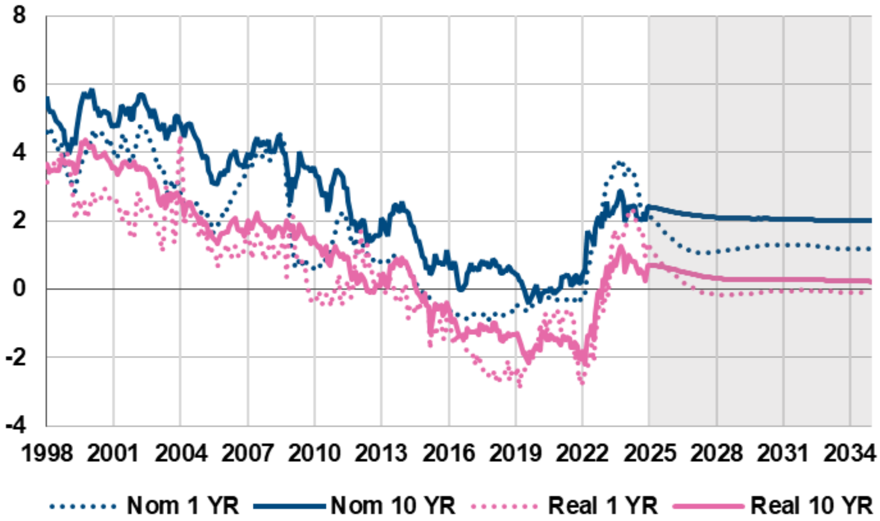
Figure 1 shows historical interest rates (as defined in Equation 4) up to and including 2024 and then an average of the simulated interest rates according to the framework. The interest rate trend declined in the 2000s but increased in conjunction with higher inflation outcomes in 2022. The one-year rate has historically often been lower than the ten-year rate. An exception is 2022 when the short-term interest rate rose in conjunction with high inflation and thereby exceeded the long-term interest rate. The short rate also exhibits larger fluctuations than the long rate, which is in line with the fact that short rates are more volatile than long rates over time.

The real rate exhibits, as does the nominal rate, a downward trend. Since holders of inflation-linked bonds receive compensation for inflation outcomes, it is reasonable that the difference between ten-year nominal and real rates have historically oscillated at around two per cent, which is the Riksbank’s inflation target.

It is also evident in Figure 1 that the simulated ten-year nominal rate is above the one-year rate, which is in line with the historical pattern. The difference is just below one percentage point.

Figure 1 Interest rates according to framework

Per cent



Note: The figure shows one-year and ten-year nominal- and real interest rates (defined as zero-coupon yields). Historical interest rates up to 2024 are provided by the estimated interest rate parameters according to the Nelson-Siegel model. Interest rates as of 2025 are the average of 10,000 simulated paths according to the DNS model described in the above chapter.

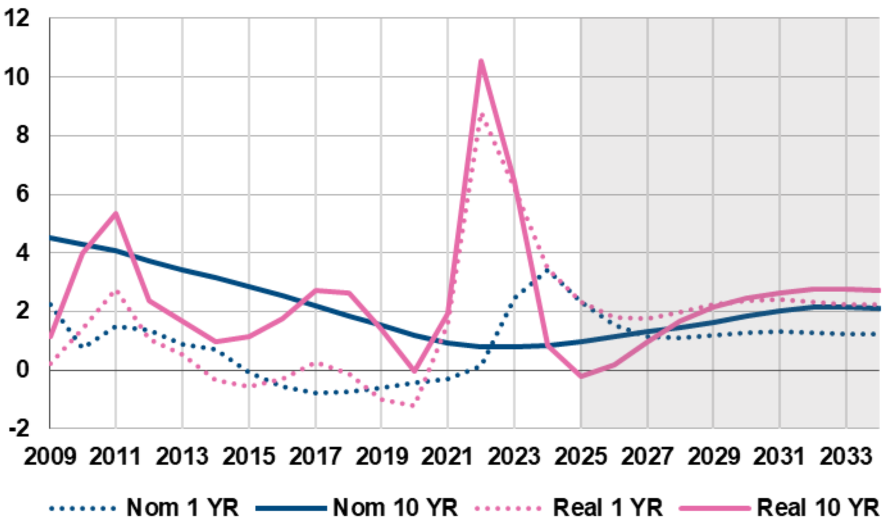
Source: The Debt Office

With the aid of the interest rates, we calculate the costs for funding strategies with individual instruments (as defined in Equation 6). Figure 2 shows four of these strategies consisting solely of one-year nominal bonds, ten-year nominal bonds, one-year inflation-linked bonds, and ten-year inflation-linked bonds, respectively.[10]

[10] Issuing a one-year inflation-linked bond has not been part of the Debt Office's borrowing strategy. This report outlines the characteristics of funding strategies across various terms to maturity.

Figure 2 Cost based on modelled interest rates

Per cent



Note: The figure shows costs, in per cent of total debt, for selected individual-instrument funding strategies, i.e. portfolios consisting of instruments with the same term to maturity and type (nominal or inflation-linked bonds). Nom 1 YR shows, for example, the cost of the strategy whereby one-year nominal bonds are issued evenly over time. The grey area is the annual average cost based on simulated average interest rates.

Source: The Debt Office

The costs for nominal funding strategies with long maturities are less volatile than those with short ones because a lower share of the debt is refixed at a new interest rate every month. Figure 2 shows, for example, that the historical trend for the cost of a funding strategy with ten-year nominal bonds is much more even than a strategy with one-year nominal bonds. The cost for funding strategies with inflation-linked bonds exceeds that for nominal bonds during 2022 and 2023 when inflation was high. Since inflation-linked bonds compensate investors for high inflation, the Debt Office's cost as an issuer increases with inflation.

Table 1 Cost and risk for funding strategies with individual instruments in 2025

Per cent

Strategy med individual instrument	Cost (total)	Risk (total)	Cost (real interest rate)	Cost (inflation)	Risk (real interest rate)	Risk (inflation)
Nom 1 YR	1.4	1.8				
Nom 2 YR	1.5	1.8				
Nom 3 YR	1.7	1.7				
Nom 4 YR	1.7	1.6				
Nom 5 YR	1.8	1.4				
Nom 6 YR	1.8	1.3				
Nom 7 YR	1.8	1.2				
Nom 8 YR	1.8	1.1				

Strategy med individual instrument	Cost (total)	Risk (total)	Cost (real interest rate)	Cost (inflation)	Risk (real interest rate)	Risk (inflation)
Nom 9 YR	1.7	0.9				
Nom 10 YR	1.7	0.8				
Infl.-linked 1 YR	2.2	2.1	0.1	2.0	1.2	0.9
Infl.-linked 2 YR	2.1	2.2	0.1	2.0	1.2	1.0
Infl.-linked 3 YR	2.1	2.2	0.1	1.9	1.1	1.1
Infl.-linked 4 YR	2.0	2.2	0.1	1.9	1.0	1.2
Infl.-linked 5 YR	2.0	2.2	0.1	1.9	0.9	1.3
Infl.-linked 6 YR	1.9	2.1	0.0	1.9	0.7	1.4
Infl.-linked 7 YR	1.9	2.1	0.0	1.9	0.6	1.5
Infl.-linked 8 YR	1.9	2.1	0.0	1.9	0.5	1.6
Infl.-linked 9 YR	1.8	2.0	-0.1	1.9	0.4	1.7
Infl.-linked 10 YR	1.8	2.0	-0.1	1.9	0.3	1.7

Note: The cost is an annual average of interest costs for the individual-instrument funding strategies. The horizon for calculations is ten years from 2025 until 2034. The risk measure is rES and measures the difference between high cost and average cost according to the description in the above chapter. Cost and risk for inflation-linked bonds are divided into two components. The first is cost attributed to the real interest rate, and the other is cost attributed to inflation compensation.

Source: The Debt Office

Table 1 shows the cost and risk for all funding strategies with nominal and inflation-linked bonds with maturities from one to ten years. Among the strategies with nominal bonds, a longer maturity entails higher cost and lower risk. The risk is lowered from 1.8 per cent to 0.8 per cent when the maturity for a funding strategy increases from one-year to ten-year bonds while the cost increases from 1.4 per cent to 1.7 per cent. The reduction in risk is due to the interest rate being refixed less often for debt with a long term to maturity and that the short rate is more volatile than the long rate. The increase in cost is consistent with a positive term premium.

The cost for inflation-linked bonds can be divided into two components. The first is cost attributed to the real interest rate, and the other is cost attributed to inflation compensation. The cost attributed to the real rate is close to zero per cent for all maturities. Longer maturities, however, provide lower risk for this component.[11] The other component – cost due to inflation – is around two per cent regardless of maturity, but the risk increases for longer maturities. Inflation-linked bonds thereby entail a higher total cost than nominal bonds, a factor that is mainly driven by inflation compensation.

[11] This is consistent with Ang et al. (2008) who show that the difference between different maturities for the US real interest rate is small. The variation in the short-term interest rate is, however, greater than in the long-term interest rate.

Debt portfolios with different terms to maturity and compositions provide basis for analysis of cost and risk

We construct hypothetical portfolios with different terms to maturity and compositions. By using the framework described above, we can analyse cost and risk for the hypothetical portfolios. The results of comparing the portfolios can be applied to balancing cost and risk in managing the central government debt.

Two types of debt portfolios are presented in the analysis. The first type is those hypothetical portfolios that reflect the Debt Office's current strategy. The other type is created through optimisation and shows compositions that have the lowest cost with a given risk. The optimised compositions are collectively termed the efficient frontier.

We start with the first type of composition and construct hypothetical portfolios with different proportions of short and long bonds, which gives the portfolios different maturities. In addition, we analyse a portfolio with a higher inflation-linked share. Details about the compositions are presented in Table 2. Portfolio 3 (target value) represents in broad terms the Debt Office's current strategy. The portfolio's composition is achieved by issuing a large share of ten-year bonds and a smaller share of bonds with maturities of five and two years. This also includes instruments with a maturity of one year (corresponding to treasury bills) and inflation-linked bonds. Portfolio 1 (short) provides a shorter maturity whereas portfolio 5 (long) provides a longer maturity through variation in the share of one-year nominal bonds and ten-year nominal bonds.

Table 2 Composition of the hypothetical portfolios with different maturities

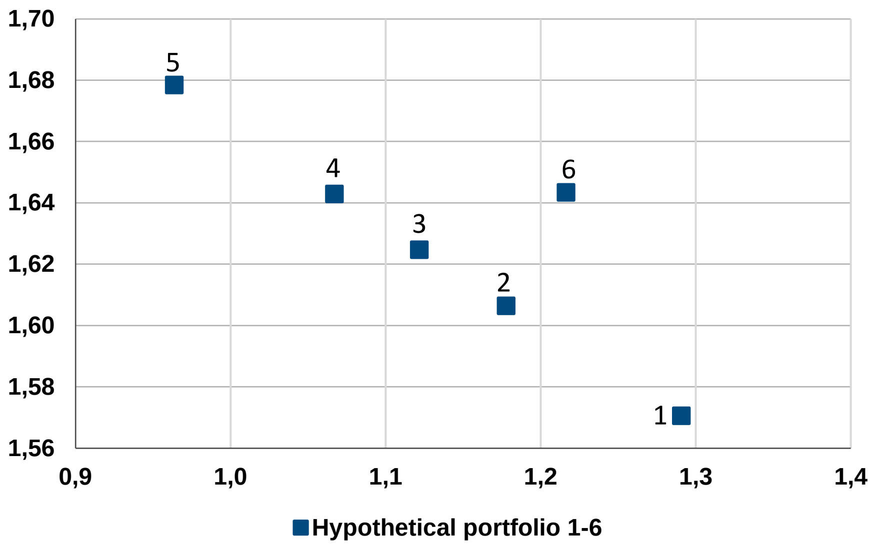
Portfolio	Nom 1YR	Nom 2YR	Nom 5YR	Nom 10YR	Infl.-linked 10YR	Term to maturity
1 (short)	39%	3%	7%	41%	10%	2.94
2	27%	3%	7%	53%	10%	3.47
3 (target value)	21%	3%	7%	59%	10%	3.74
4	15%	3%	7%	65%	10%	4.01
5 (long)	4%	3%	7%	76%	10%	4.54
6 (infl.-linked)	21%	3%	7%	45%	24%	3.74

Note: The rows show six hypothetical portfolios that are achieved by issuing one-year, two-year, five-year, and ten-year nominal bonds as well as ten-year inflation-linked bonds. Term to maturity is interest rate refixing period and expressed in years. Portfolio 3 (target value) has, for example, a term to maturity of 3.74 years according to the model and corresponding to 4.75 years when taking into account that the Debt Office has ultra-long bonds. The Debt Office currently has three ultra-long bonds outstanding, the maturities of which were 25, 30, and 50 years on issuance. These constitute a small part of the debt and are excluded in the analysis.

Source: The Debt Office

Figure 3 Expected cost and risk for hypothetical portfolios 2025–2034

Per cent



Note: The figure shows expected cost and risk for six of the hypothetical portfolios. The Y axis shows average cost and the X axis shows rES. Both measurements are shown as percentage of total debt and calculated according to the previous description.

Source: The Debt Office

Figure 3 shows that the cost increases from portfolio 1 to portfolio 5 but the risk decreases. A longer maturity involves a higher cost but lower risk. Despite having the same maturity, portfolio 6 has a higher cost and risk than portfolio 3. Inflation-linked bonds are thereby not effective in terms of cost and risk in our analysis.

Table 3 shows cost and risk for the portfolios with the assumption that the size of the central government debt remains SEK 1,100 million for ten years forward. Cost and risk are expressed in billions of kronor in the table instead of per cent as in Figure 3. The findings confirm the conclusion that longer maturities involve higher expected cost and lower risk. The differences between the various maturities are nevertheless small in relation to the model’s uncertainty.

Table 3 Cost and risk for hypothetical portfolios with different maturities in 2025

SEK billion

Portfolio	Cost	Risk
1 (short)	17.3	14.2
2	17.7	13.0
3 (target value)	17.9	12.3
4	18.1	11.7
5 (long)	18.5	10.6
6 (infl.-linked)	18.1	13.4

Note: Cost is the average cost per year based on 10,000 simulations between 2025 and 2034. Risk shows an extra cost in unfavourable scenarios compared with the cost on average. Unfavourable scenarios refer to the 500 simulations with the highest cost. In these scenarios, the cost exceeds the 95th percentile of 10,000 simulations.

Source: The Debt Office

The other type of portfolio, those that make up the efficient frontier, is defined by the composition that provides the lowest cost with a given risk. Table 4 shows the composition as well as the cost and risk for these portfolios. Figure 4 then illustrates cost and risk for the efficient frontier. Portfolios 1-6 according to Table 2 are also included for comparison.

Cost and risk for the efficient frontier in Table 4 confirm that a shorter maturity involves a higher cost but lower risk. All portfolios are composed of one-year and ten-year nominal bonds. The portfolio that provides the lowest cost, given the sub-condition that risk cannot for instance exceed one per cent is portfolio 9, which consists of 20 per cent in one-year nominal bonds and the remainder in ten-year nominal bonds. The share of inflation-linked bonds is zero as they neither lower the cost nor the risk.

Table 4 Portfolios that constitute an efficient frontier 2025-2034

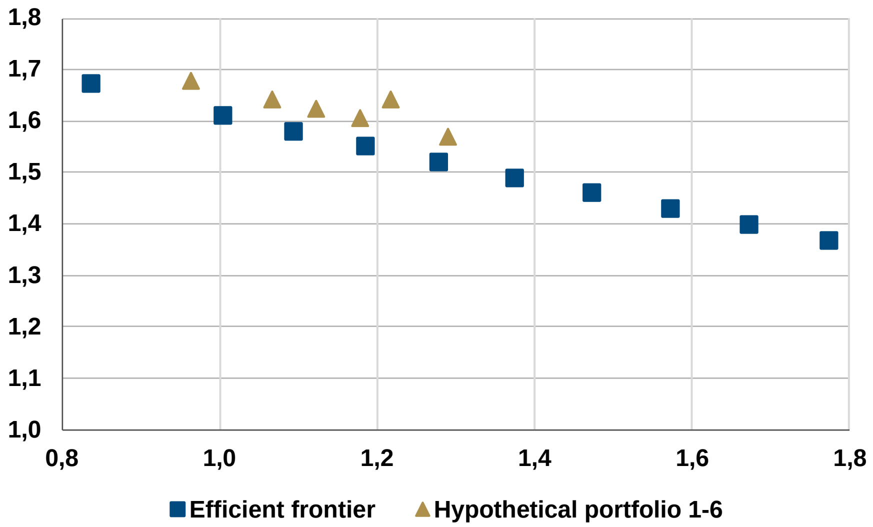
Portfolio	Cost	Risk	Nom 1 YR	Nom 10 YR	Term to maturity
1	1.37%	1.77%	100%	0%	0.5
2	1.40%	1.67%	90%	10%	0.9
3	1.43%	1.57%	80%	20%	1.4
4	1.46%	1.47%	70%	30%	1.8
5	1.49%	1.38%	60%	40%	2.3
6	1.52%	1.28%	50%	50%	2.7
7	1.55%	1.18%	40%	60%	3.2
8	1.58%	1.09%	30%	70%	3.6
9	1.61%	1.00%	20%	80%	4.1
10	1.67%	0.84%	0%	100%	5.0

Note: The table shows ten portfolios with maturities varying between 0.5 and 5 years. The portfolios have the lowest cost with the risk involved and are therefore in the efficient frontier. Nom 1YR and Nom 10 YR show the proportions of one-year nominal bonds and ten-year nominal bonds in the target-value portfolios. Nominal bonds with maturities between two and nine years as well as inflation-linked bonds with maturities between one and ten years are not shown in the table because all weights for the bonds are zero.

Source: The Debt Office

Figure 4 Efficient frontier and hypothetical portfolios 2025–2034

Per cent



Note: The figure shows cost and risk for the portfolios that make up the efficient frontier and hypothetical portfolios. The Y axis shows average cost and the X axis shows rES. Both measurements are shown as percentage of total debt and calculated according to the previous description.

Source: The Debt Office

Figure 4 shows that the hypothetical portfolios are just above the efficient frontier. Portfolios 1-5 are closer to the efficient frontier than portfolio 6 is. The larger inflation-linked share in portfolio 6 contributes to a higher expected cost given a risk level of just above 1.2 per cent.

In summary, the analysis shows that a longer term to maturity involves a higher cost but lower risk. Inflation-linked bonds neither contribute to a lower cost nor a lower risk for the debt. According to the simulation results from 2025, only combinations of one-year and ten-year nominal bonds are effective, since the compositions of those two provide the lowest cost for a given level risk.

Comparisons with other methods

The term premium is an important factor to consider when managing the maturity of the central government debt. This premium measures the difference between a long rate, such as a ten-year yield, and the expected short rate in the same period. The ten-year term premium captures the compensation that investors demand for investing in a ten-year bond compared with investing in a short-term bill and then rolling the position for ten years. For the central government, loans with longer maturities entail reduced risk in terms of cost variation for the central government debt. At the same time, higher term premia have meant that the central government is expected to pay higher prices for borrowing in longer maturities.[12] There are different models for estimating term premia and the estimates differ. What they have in common though is that they show a declining trend for many developed countries in recent decades.[13] This trend was broken after 2020, but the term premium still remains at a historically low level.

As a part of the basis for the decision on steering the debt's term to maturity, the Debt Office has historically calculated the ten-year term premium according to the what is known as the ACM model.[14] Our simulation framework also implies a ten-year term premium since, with the aid of the DNS model, we can calculate the average of the simulated short rates ten years forward. The ten-year term premium is, according to our framework, the difference between the observed ten-year rate at a certain point in time and the average of future short rates. We make this calculation for every month as of 2015. This enables us to examine whether the ACM model and DNS model have provided different estimates historically.

Figure 5 shows a ten-year average of the expected future short rates for ACM and DNS. The expected short rate for ACM is persistent over time and varies between just below zero per cent and two per cent. The expected short rate according to DNS varies much more. During the 2023–2024 period, the two methods provide similar results. Between 2015 and 2022, the estimates differ. DNS provides negative short rates whereas ACM provides an expected short rate just above zero. The expected short rate according to DNS is very high at above 11 per cent at the end of 2022 so, for that time, the ACM provides a more reliable estimate.

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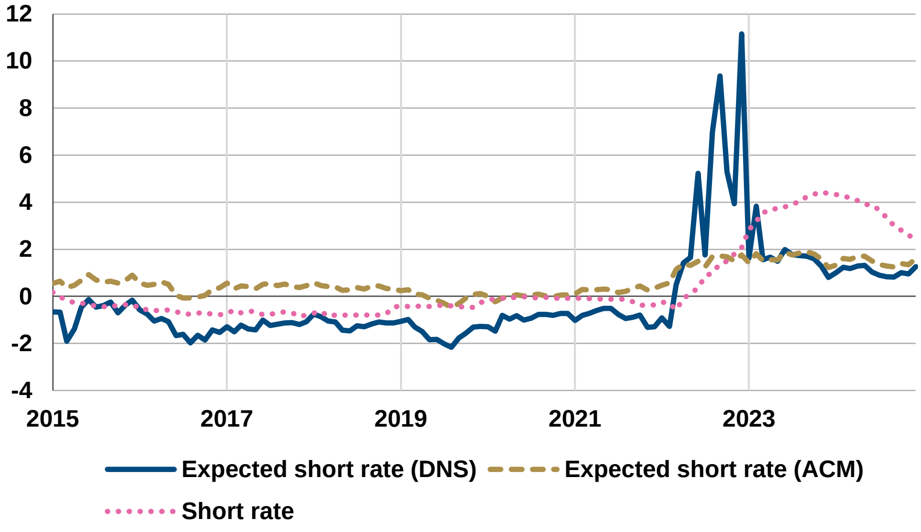
[12] The decrease in risk with the debt's term to maturity mostly applies to maturities up to and including ten years. According to simulations in Belton et al. (2018), for example, the risk for the US central government increases if the term to maturity is extended from ten to 20 or 50 years. This is because longer maturities are associated with higher cost, which over time also increases the size of the central government debt. The larger debt in turn brings with it greater cost volatility for longer maturities.

[13] The trend applies to, among others, the US, Germany, Australia, and Japan. See Cohen et al. (2018), Jennison (2017), and Tang et al. (2019) for details.

[14] The ACM model was developed by Adrian et al. (2013). See the Debt Office's proposed guidelines for 2025–2027 for an example of the Swedish term premium.

Figure 5 Short rates according to DNS and ACM

Per cent



Note: The figure shows the expected short rate, in per cent, estimated with the DNS and ACM models. The frequency is monthly from January 2015 to December 2024. The expected short rate according to DNS is calculated as an average of the monthly interest rate ten years forward. The data for estimating the term premium according to ACM covers October 1997 through December 2024. “Short rate” shows the outcome of the monthly rate according to the NS model.

Source: The Debt Office

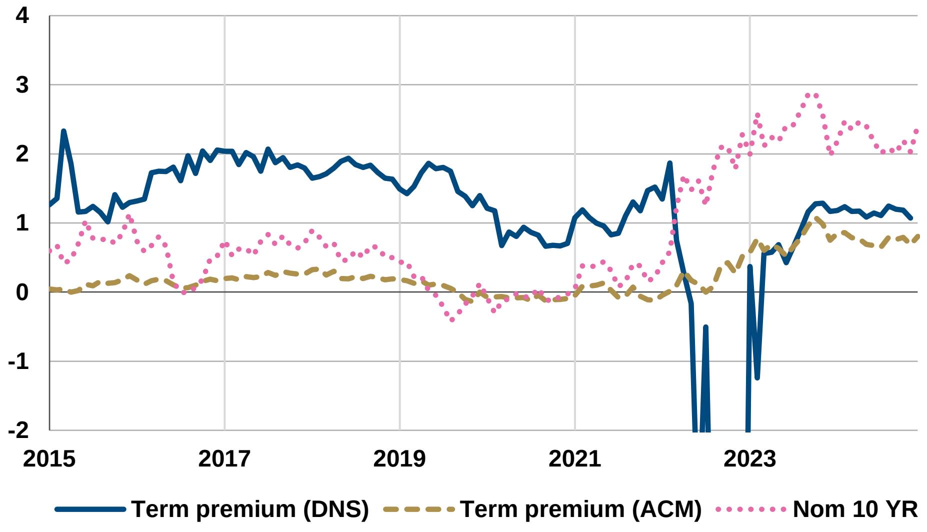
Figure 6 shows the ten-year term premium calculated according to ACM and our DNS model. Since the expected short rate according to ACM is slightly higher than that for DNS, the term premium according to ACM is slightly lower. As previously discussed, DNS provides an unreasonably high expected short rate at the end of 2022, which leads to a very negative term premium. This problem is, however, temporary. Both the term premium and the ten-year rate increased slightly after 2022.[15]

Both models provide estimates of the same magnitude for the term premium after 2022. The calculation of cost and risk for different portfolios in the above chapter is based on December 2024. Therefore, the historical differences between the models do not affect the current results for cost and risk.

[15] See for example the discussion in chapter 34 of Rebonato (2018). Changes in term premia have a tendency to be driven by changes in the ten-year interest rate rather than the expected short rate.

Figure 6 Term premium according to DNS and ACM

Per cent



Note: The figure shows the ten-year term premium estimated with the DNS and ACM models. The frequency is monthly from January 2015 to December 2024. The data for estimating the term premium according to ACM covers October 1997 through December 2024. The term premium is calculated once per month as of 2015 forward. To better illustrate the curves, the Y axis is limited to -2 per cent. The term premium decreases though to -9 per cent at the end of 2022.

Source: The Debt Office

The Debt Office has previously analysed the debt's composition by measuring the difference between break-even inflation and expected inflation. This method measures the expected cost savings between issuing inflation-linked bonds and nominal bonds with the same maturity. The framework with portfolio simulations can also measure the expected cost difference between issuing inflation-linked bonds instead of nominal bonds. The difference between the methods is mainly in regard to how the inflation expectation is derived. One uses questionnaire surveys whereas the other uses a statistical model. The two methods provide similar results after 2023.

Just as the DNS model provides high estimates for future short rates in 2022, it also provides high estimates for inflation. Inflation according to the model was expected to increase from a level of 12 per cent in December 2022. This is much higher than indicated by the surveys at the time. Then, surveys indicated a rate of inflation of three and a half per cent on average in the next five years.

We therefore conclude that the DNS model is sensitive to what historical time period is used in the estimation. If the results of the DNS model were to be incongruous or deviate significantly from other models, that should be resolved. One way is to combine the DNS model with surveys, sometimes called anchoring, in order to anchor the parameters to certain values.[16] Another way might be to

[16] A more thorough description can be found in Altavilla et al. (2014).

manually build different trends of future interest rates and inflation instead of the VAR model used in DNS.

The DNS model is not intended to replace the ACM model for estimating the term premium, which is a challenge to calculate. We conclude that the DNS model's estimates are at the same level as those of the ACM model right now, and what the model then adds is an intuitive analysis of the cost variations within the same framework.

Altogether, this indicates a need for further analysis that weighs together results from different models and evaluation periods to form a comprehensive assessment of the balance between cost and risk.

Evaluation period plays role for model results

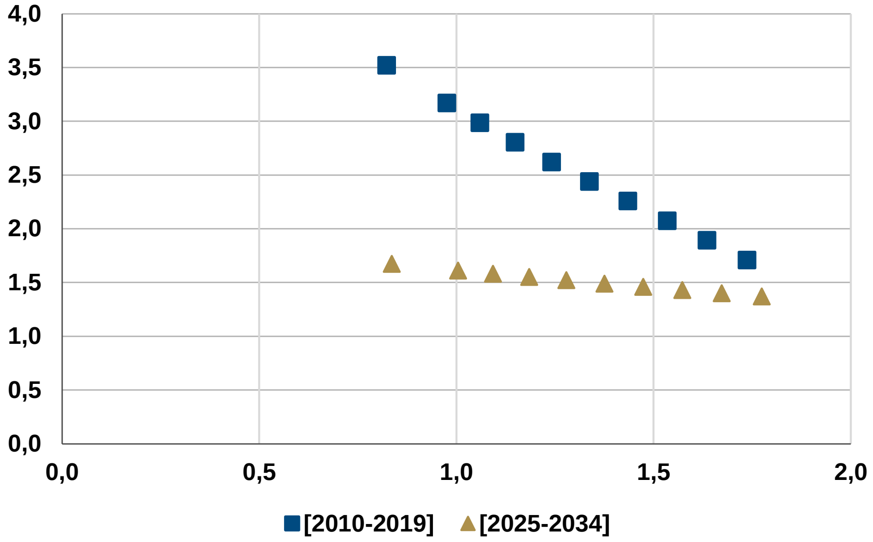
In this chapter, we study how the choice of evaluation period affects the results. As an alternative evaluation period, we choose the 2010–2019 period. The data used in the analysis ends in December 2009 and the simulations start with January 2010. This period builds on historical interest rates that are significantly higher than those that are observed later. The yield curve is also steeper, which means that differences between long and short rates were greater in 2009.

Figure 7 shows the efficient frontier based on results for the years 2010–2019. The previous results for the years 2025–2034 are also included in the figure. The results for the years 2010–2019 show a declining trend and confirm that longer-maturity portfolios have a higher cost and lower risk. The declining trend – the slope of the curve – is stronger than the efficient frontier for the years 2025–2034. The cost difference is in other words larger then, which is line with the term premium having been higher historically. It is also evident in the figure that all portfolios for 2025 are above those from 2010, but the differences are greater for portfolios with a shorter maturity.

The risk level for the efficient frontier is at the same level between the two time periods. The risk measured as an additional cost in unfavourable scenarios with higher interest rates varies between 0.75 and 1.75 per cent for both periods. According to the model, the cost difference has varied more from one time period to another, whereas the difference in risk has been stable. This means that a shorter maturity provides a greater cost reduction for every unit of risk based on the 2010–2019 period than the 2025–2034 period. The fact that the difference in results between the different time periods is relatively large may prove significant for decision makers.

Figure 7 Efficient frontier for different evaluation periods

Per cent



Note: The figure shows the efficient frontier according to the DNS model for different time periods. The first is based on historical data between October 1997 and December 2009, and with simulations for the 2010–2019 period. The second is based on data between October 1997 and December 2024, and with simulations for the 2025–2024 period. The Y axis shows average cost and the X axis shows rES. Both measurements are shown as percentage of total debt and calculated according to the previous description.

Source: The Debt Office

Conclusions

This Focus Report presents a framework for evaluating cost and risk for different maturities and compositions of the central government debt. The findings confirm a well-known fact regarding term to maturity, which is that long maturities are associated with higher cost but lower risk. The framework helps depict these differences in figures.

We also identify an efficient frontier consisting of the portfolios that have the lowest cost with a given level of risk. Despite the fact that the portfolios are a theoretical construction without accounting for whether they are achievable in practice, the framework contributes to the analysis of the balance between cost and risk that is crucial for managing the central government debt.

Based on the current evaluation period, the differences in both expected cost and risk between the different maturities are small. Based on the historical evaluation period, between 2010 and 2019, the differences in the expected costs are greater. The choice of evaluation period does matter, and the results may therefore change in the future on the basis of new data and conditions.

The framework is based on different assumptions that are important to have in mind when implementing it. This merits further development. Certain assumptions may need to be adapted to the issue being examined or to new market conditions. We also note that the model is sensitive to the rapid increases in inflation and the short-term interest rate that occurred in 2022. This entailed that the term premium then rapidly decreased, to subsequently recover. As of 2023, according to the framework, the term premium is at the same level as that according to the ACM model, which is an alternative method used by the Debt Office.

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Appendix 1: Calculation of costs for inflation-linked bonds

Nominal bonds with a fixed coupon provide investors with a fixed amount every year regardless of inflation outcomes. Inflation-linked bonds, however, produce a higher yield if inflation is high. To calculate the cost, a continual revaluation of inflation is applied to an inflation-linked bond. The revaluation is done for both the coupon and nominal amount.

The costs between month *t* and *t*+1 for an inflation-linked bond with par yield *c* and nominal value of one krona are the difference in accrued costs between months:

$$K_{i,n} = \underbrace{\frac{I_t}{I_b} * c_{i.n} * \left(\frac{I_{t+1}}{I_t} * \tau_{t+1} - \tau_t\right)}_{\text{Change i accrued coupon}} + \underbrace{\frac{max\,(I_{t+1}, I_b) - max\,(I_t, I_b)}{I_b} * 1}_{\substack{\text{Change in accrued inflation} \\ \text{compensation on nominal amount}}}$$

Where I_t (I_{t+1}) is CPI-lagged by three months at month t $(t+1)$. I_b is the bond's base index that is measured at the time of issue, $c_{i,n}$ is par yield, and τ_t is expressed in years and is the length of time between month *t* and when the most recent coupon payment was made (i.e. $\tau_t = \frac{t}{12}, \tau_{t+1} = \frac{t+1}{12}$).[17]

We adapt the compensation so that the CPI used to adjust inflation for the nominal amount is not lower than the base index. This is to take into account the deflation protection that the Debt Office offers investors.

[17] Below is an example where we calculate the cost for January 2024 ($I_t = 408$) for a bond issued in January 2020 ($I_b = 336$) with par yield c at 2 per cent. Otherwise, the coupon payment occurs in December so $\tau_t = 1/12$ and $\tau_{t+1} = 2/12$ and that $I_{t+1} = 409$. The cost is then: $\frac{408}{336} * 2\% * \left(\frac{409}{408} * \frac{2}{12} - \frac{1}{12}\right) + \frac{409-408}{336} = 0.5\%$

The Swedish National Debt Office is the central government financial manager and the national resolution and deposit insurance authority. The Debt Office thus plays an important role in the Swedish economy as well as in the financial market.

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