This paper studies a banking model of maturity transformation in which regulatory arbitrage induces the existence of unregulated shadow banking next to regulated commercial banking. In our model, banking optimally relies not only on demand deposits but also on wholesale funding. We first show that regulatory arbitrage may set the stage for runs in the shadow banking sector. Second, we emphasize a novel channel through which these runs are contagious, affecting the regulated banking sector: Because a run on shadow banks induces a fire-sale, funding conditions in the wholesale funding market may deteriorate also for regulated banks. This creates real costs for regulated banks, and depending on the size of the shadow banking sector, it may lead to insolvency of the regulated banking sector. We use our model to argue that regulatory arbitrage poses a real threat to financial stability beyond explicit or implicit contractual linkages between regulated and non-regulated banking, and we discuss further regulatory implications such as restrictions on wholesale funding.
1. Introduction

In this paper, we show that regulatory arbitrage may set the stage for panic-based runs in the shadow banking sector, which may affect the regulated banking sector via a deterioration of overall funding conditions. We thereby emphasize a novel channel for how regulatory arbitrage in banking may contribute to financial instability. Regulated banking maybe affected by non-regulated banking via channels that go beyond explicit or implicit contractual linkages. We argue that if a regulator has the objective to shield regulated financial institutions from financial instability altogether, restrictions on wholesale funding with associated allocative inefficiencies may be necessary.

Previous to the 07-09 financial crisis, many commercial banks had set up off-balance sheet conduits to finance long-term real investment by issuing short-term debt. In the summer of 2007, increased delinquency rates on subprime mortgages ultimately led to the collapse of the conduits’ main source of funding: the market for asset-backed commercial papers (ABCP) (see, e.g., Kacperczyk and Schnabl, 2009; Covitz et al., 2013). Many commercial bank had explicitly or implicitly sponsored these conduits, and the collapse forced those banks to take the asset backed securities (ABS) and the associated risks on their balance-sheets, creating severe solvency issues.

From an ex-post perspective, it has become more or less undisputed that off-balance sheet banking had to a large extent been conducted to circumvent existing capital regulation (see, e.g., Acharya et al., 2013). In this context, the adverse implications of direct or indirect contractual linkages between regulated and unregulated banks have been identified as a particularly important source of instability (Segura, 2014). Consequently, the overwhelming regulatory response has been to close the obvious loopholes in regulation by outright prohibition of contractual links between depository institutions and other parts of the financial system (compare, e.g., Section 619 of the Dodd-Frank Act referred to as “Volcker Rule”, Report of the Vickers Commission, Liikanen Report).

In this paper, we study a banking model of maturity transformation in which intermediaries optimally rely on wholesale funding: Banks optimally do not store goods in order to manage the liquidity needs of their depositors, but rely on liquidity provided from outsiders. We first show that in the absence of deposit insurance, intermediaries are disciplined by short-term debt holders, but panic-based runs may take place. We then show that while a deposit insurance may prevent panic-based runs, it also gives rise to moral hazard as it undermines the disciplining effect of short-term debt. The resulting moral hazard makes costly regulation necessary, which in turn gives incentives

---

1To some observers, this had already been clear prior to the crisis, see Jones (2000).
to circumvent regulation. This induces the coexistence of a regulated banking sector next to an unregulated shadow banking sector. The regulated banking sector is covered by a deposit insurance and thus not exposed to panic-based runs. In turn, in the shadow banking sector, short-term debt is disciplining, but panic-based runs may occur.

Given that intermediation optimally relies on wholesale funding, regulatory arbitrage poses a real threat to overall financial stability. Particularly, also to the stability of regulated banks is endangered, even in the absence of any contractual linkages to shadow banks. The main mechanism is that a panic-based runs in the shadow banking sector will lead to fire-sales that may induce cash-in-the-market pricing, and consequently to a deterioration of market funding conditions for regulated banks that rely on such funding as well. This creates real costs for commercial banks and potentially leads to their illiquidity and insolvency. Moreover, these costs grow with the size of the shadow banking sector, as a larger shadow banking sector induces stronger fire-sale effects. This indicates that if one has the objective to shield regulated banks from financial instability altogether, it may be necessary to implement further restriction on a commercial banks liability side. In particular, restrictions on wholesale funding for depository institutions may be necessary, inducing allocative inefficiencies.

The understanding of such kind of mechanisms is particularly important in the light of the new regulations implemented in the aftermath of the 07-09 financial crisis. Many of the new regulatory measures have been implemented under the premise that a prohibition of explicit or implicit contractual linkages between depository institutions and other types of banking can ensure financial stability. Less emphasis has been put on the question whether depository institutions should also be allowed to partially use market funding as a source of funding. Our paper allows us to shed new light on this question. We argue that prohibiting contractual linkages is not sufficient to shield the regulated banking sector from financial fragility. We show that restricting wholesale funding in depository institutions may be necessary to shield the regulated banking sector in the presence of regulatory arbitrage. However, this may in turn increase allocative inefficiencies and lead to a growth of fragile shadow banking.

Our model builds on – and at the same time nests – the model by Diamond and Dybvig.

---

2In particular, much focus has been of prohibiting sponsor support. Moreover, the separation of regular banking proprietary trading has often been targeted. Little attention has been paid to the liability side of deposit banks.

3E.g., the Financial Services Act of 2013 which was based in the Vickers Commissions Report limits the exposure of depository institutions to other financial institution within the same bank holding company. It assumes that ring-fencing can be useful to shield the ring-fenced part of institutions from costs of financial distress. It is acknowledged that wholesale funding displays a risk, but does not unambiguously recommend restrictions.
We enrich the setup along two dimensions: On the one hand, we introduce a new type of agents, called “investors”. These agents are only present from the interim period onwards and can provide funds to the bank in exchange for claims on future cash-flows. This makes an arrangement optimal that we refer to as “wholesale funding”, and the use of storage becomes inefficient. On the other hand, we introduce a shirking technology that allows to emphasize a disciplining role of short-term debt in the sense of Calomiris and Kahn (1991) and Diamond and Rajan (2001). First, we show that short-term debt is disciplining in our setup and thus allows intermediaries to implement the first-best allocation. Intermediaries will always refrain from shirking as they do not benefit from it if their depositors run. We also show, however, that the disciplining effect of short-term debt is necessarily associated with the possibility of panic-based runs. A regulator that decides to provide a deposit insurance to eliminate panic-based runs would thus undermine the disciplining effect of short-term debt. This makes it necessary to complement a safety net with additional regulatory measures. In our setup, regulation can induce diligent behavior via a textbook skin-in-the-game mechanism à la Tirole (2010).

We assume that regulatory arbitrage allows for the emergence of unregulated “shadow banks” next to the regulated “commercial banks”. Institutions in this non-regulated banking sector can experience panic-based runs because they are not covered by the deposit insurance. We emphasize a novel theoretical channel through which these runs may be contagious, affecting the regulated banking sector: A run on shadow banks induces a fire-sale with cash-in-the-market pricing à la Allen and Gale (1994) and Martin et al. (2014b). This implies that the funding conditions for regulated banks that rely on wholesale funding deteriorate, thus creating real costs for regulated banks. Depending on the size of the shadow banking sector, it may ultimately lead to the insolvency of regulated banks, making the provision of a deposit insurance costly in equilibrium. This contagion channel can be shut down by restrictions on wholesale funding. These restrictions do however lead to allocative inefficiencies and increase the shadow banking sector.

Finally, we also briefly discuss the role of direct contractual linkages between the two sector in the form of liquidity guarantees. We show that liquidity guarantees are optimal from the perspective of a single institution, but increase the parameter space in which runs in the shadow banking sector are systemic and contagious, and exacerbate the adverse consequences of runs. This shows that the prohibition of contractual linkages is indeed desirable. We argue, though, is is not sufficient to shield regulated banks from financial instability.
Our paper reaches out to the recent literature on shadow banking, especially to the fast-growing literature on theoretical aspects of shadow banking. Our modeling approach is related to the paper by Martin et al. (2014a). Their focus lies on the run on repo, and on the differences between bilateral and tri-party repo in determining the stability of single financial institutions. In turn, runs in our model are more closely related to runs on ABCP and system-wide crises. The paper by Segura (2014) discusses the role of liquidity guarantees. We show that there may be reasons to believe that regulatory arbitrage may affect stability of regulated banks beyond such contractual linkages. The paper by Bolton et al. (2011) is the first contribution to provide an origination and distribution model of banking with multiple equilibria in which adverse selection is contagious over time. Gennaioli et al. (2013) provide a model in which the demand for safe debt drives securitization. In their framework, fragility in the shadow banking sector arises when tail-risk is neglected.

Other contributions that deal with shadow banking are Ordoñez (2013), Goodhart et al. (2012, 2013), and Plantin (2014). Ordoñez focuses on potential moral hazard on the part of banks. In his model, shadow banking is potentially welfare-enhancing as it allows to circumvent imperfect regulation. However, it is only stable if shadow banks value their reputation and thus behave diligently; it becomes fragile otherwise. The emphasis of Goodhart et al. lies on incorporating shadow banking into a general equilibrium model. Plantin studies the optimal prudential capital regulation when regulatory arbitrage is possible. In contrast to all three, we focus on the destabilizing effects of shadow banking in the sense that it gives rise to run equilibria.

2. Setup

Consider an economy that goes through a sequence of three dates, $t \in \{0, 1, 2\}$. There is a single good that can be used for consumption as well as for investment. The economy is populated by three types of agents: consumers, intermediaries and investors.

Technologies

Altogether, there are three types of technologies available for investment (see a summary of the payoff structure in Table 1). There is a short technology (“storage”) available in $t = 0, 1$, transforming one unit invested in $t$ into one unit in $t+1$. Moreover, there are two illiquid technologies available for investment in $t = 0$: a “productive technology” and an unproductive “shirking technology”. Both technologies are assumed to be technologically illiquid, i.e., for one unit invested they produce a cash-flow $\ell$ in $t = 1$ only if they are physically liquidated, and the physical liquidation rate of the technologies is assumed to
be $\ell \to 0$. Note that the technologies (or claims on the technologies’ future returns) may nonetheless be sold at higher values at a secondary market that will be specified below.

The return properties of the illiquid technologies in $t = 2$ are as follows:

- **Productive technology**: One unit invested yields a safe return of $R$ units in $t = 2$.

- **Shirking technology**: One unit invested yields a safe return of $R_{\text{shirk}} < 1$ in $t = 2$. However, the technology yields a private benefit $B > 0$ in $t = 2$. The private benefit is available only to the agent who conducted the initial investment, i.e., it is non-transferable and non-contractible. Moreover, it only accrues if the technology is not liquidated in the interim period.

<table>
<thead>
<tr>
<th></th>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage in $t = 0$</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Storage in $t = 1$</td>
<td>0</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Productive technology</td>
<td>-1</td>
<td>$\ell \to 0$</td>
<td>$R$</td>
</tr>
<tr>
<td>Shirking technology</td>
<td>-1</td>
<td>$\ell \to 0$</td>
<td>$R_{\text{shirk}} + B$</td>
</tr>
</tbody>
</table>

**Table 1: Payoff structure of technologies**

We assume that $R_{\text{shirk}} + B \leq 1$. This implies that the shirking technology is inefficient, although it generates a private benefit. As will become clear later, the possibility of investing in this technology and financing the investment by short-term debt will give rise to moral hazard. This moral hazard will lead to the necessity of capital regulation once a deposit insurance undermines the disciplining effect of short-term debt.

**Consumers**

There is a continuum of consumers with mass one. Initially, consumers face idiosyncratic uncertainty with respect to their preferred date of consumption, and they may lend their endowment to intermediaries to invest on their behalf.

Each consumer is endowed with 1 unit of the good in $t = 0$. There are two types of consumers, *patient* and *impatient* consumers: a fraction $\pi$ is impatient and derives utility only from consumption in $t = 1$, $u(c_1)$, and a fraction $1 - \pi$ is patient and derives utility only from consumption in $t = 2$, $u(c_2)$. We restrict attention to CRRA utility, i.e., the period-utility function has the form $u(c_t) = \frac{1}{1-\eta} c_t^{1-\eta}$, with $\eta > 1$. This restriction

---

4We assume that consumers cannot invest in technologies directly in the initial stage and trade technologies in the interim period. They can only lend their funds to intermediaries. In a later section, we will argue why we can focus on a banking solution directly, i.e., why a banking solution dominates a financial markets solution in terms of welfare.
enables us to solve the model in closed form, however, the qualitative results do not depend on the CRRA assumption.

It is assumed that consumers initially do not know their type; their probability of being impatient is identical and independent, so all consumers have the same prior \( \pi \) initially. In period one, each consumer privately learns his type. This private revelation can be considered as a liquidity shock.

A consumption profile \((c_1, c_2)\) gives a consumer \(i\) an ex-ante expected utility of

\[
U(c_1, c_2) = \pi u(c_1) + (1 - \pi) u(c_2) = \frac{1}{1 - \eta} \left[ \pi c_1^{1-\eta} + (1 - \pi) c_2^{1-\eta} \right].
\]

(1)

Notice that the attributes \textit{patient} and \textit{impatient} characterize the consumer’s exogenous type which determines his preference. In contrast, the attributes \textit{late} and \textit{early} will characterize the timing of withdrawals which is endogenous: An “early consumer” withdraws in \(t = 1\), while a “late consumer” withdraws in \(t = 2\).

**Intermediaries**

There is a mass \(m \ll 1\) of intermediaries. While consumers cannot invest in the technologies directly, intermediaries face no investment restrictions. Intermediaries compete for the consumers’ funds and invest in the technologies on their behalf. Moreover, they may choose to invest their own funds.

Intermediaries only care about \(t = 2\) consumption. Each intermediary is endowed with \(E\) units of the good. We assume that \(E\) is large, implying that no result will be driven by \(mE\) becoming a binding resource constraint. Importantly, intermediaries are assumed to have an outside option, resulting in a required return of \(\rho > R\) in \(t = 2\) for each unit invested in \(t = 0\). The assumption of \(\rho > R\) will make it costly for intermediaries to use their own endowment for investment. As we will show later, this makes it costly to give intermediaries incentives to invest in the productive technology instead of the shirking technology via a skin-in-the-game mechanism.

On the liability side, intermediaries need to initially choose the deposit contract \((c_1, c_2)\) that they offer to consumers in exchange for one unit of initial deposits. Moreover, they need to choose how much inside equity \(e_0\) they want to invest in \(t = 0\), in exchange for receiving the value of the equity \(e_2\) in \(t = 2\).

On the technology side, they need to make an investment decision: We denote by \(I\) the investment in the productive technology, by \(I_{\text{shirk}}\) the investment in the shirking technology.

---

\(^5\)As the model has no aggregate uncertainty, the shape of intermediaries’ utility is not important. They might be risk-neutral or be risk-averse.

\(^6\)This assumption makes bank equity costly and is a shortcut for other frictions that make equity costly, such as adverse selection concerns Myers and Majluf (1984).
technology and by $1 + e_0 - I - I_{\text{shirk}}$, the investment in storage. We assume that the investment decision of a single intermediary is unobservable in $t = 0$, but becomes public information in $t = 1$.

**Investors**

There is a continuum of investors of mass $n$. Investors only become active in the interim period and take the role of potential liquidity providers in the interim period: Investors can transfer some of their endowment to intermediaries in exchange for a claim on some of the future cash-flows of the intermediaries’ technologies. Later, we will focus on the parameter specifications in which the existence of investors makes investment in storage inefficient. I.e. it will be optimal for intermediaries to rely on wholesale funding from outsiders instead of storing real goods. However, this will also give rise the main contagion channel between regulated and unregulated banking: When a run on shadow banks induces a fire-sale, funding conditions for regulated banks deteriorate as well.

Investors are born in $t = 1$ and receive an endowment of $A/n$, so their entire endowment is given by $A$. The endowment $A$ will be one of the crucial parameters of the model, determining whether there are multiple equilibria or not. Given that investors are born in $t = 1$, it is not possible to contract with them in $t = 0$. Investors care about consumption in period 2 are assumed to have an outside option which induces a required return of $\gamma$, where $\gamma \in [1, R]$. That is, for each unit they transfer to intermediaries in $t = 1$, they need to receive at least $\gamma$ units in $t = 2$.

There will be some price $p$ at which investors will be willing to transfer some of their endowment in exchange for a claim on the future cash flow of the intermediaries’ technologies. It is functionally equivalent in our setup directly purchases some of the intermediaries investment at price $p$, or if an investor lends to an intermediary, while taking some of the intermediary’s investment as collateral at value $p$. That is, it is functionally equivalent to conduct an outright asset sale/purchase, or to lend against collateral (wholesale funding with seniority). In most part of the analysis we will speak of asset purchase/sale. Our preferred interpretation of such kind of arrangement, is wholesale funding.

\footnote{Again, the shape of their utility function is not important as long as it is compatible with the specified outside option.}
3. Optimal Intermediation and Runs

3.1. First Best

We will now derive the allocation that maximizes the expected utility of consumers, subject to the participation and constraints of intermediaries and investors, and subject to the resource constraints. Since we are only concerned with the maximization of consumer welfare and treat the participation constraints as resource constraints, we refer to the optimal solution as the first best.

Notice first that it is optimal for impatient consumers to consume at \( t = 1 \) only, and for impatient consumers to consume at \( t = 2 \) only. For simplicity we denote an optimal allocation by \((c_1^*, c_2^*)\), where \( c_1^* \) is consumed when impatient, and \( c_2^* \) if patient.

In the first best, the shirking technology is not used, i.e. \( I_{shirk} = 0 \). In the first best, there is only investment in the productive technology, as the shirking technology’s return properties are strictly dominated. If there was positive investment in shirking, this could be substituted by an equal amount of investment in the productive technology, and additional benefits of \( R - R_{shirk} \) per unit are sufficient to compensate the intermediary for his forgone private benefit \( B \).

We denote by \( L \) the units of the productive technology that get transferred from intermediaries to investors, in exchange for \( L_p \) units of the good (“liquidity”) from investors to intermediaries in the interim period. This transaction is referred to as “wholesale funding” in the following.

The first-best maximization program is given by

\[
\begin{align*}
\max_{(c_1, c_2, e_0, e_2, I, L, p) \in \mathbb{R}^*_+} & \quad \pi u(c_1) + (1 - \pi) u(c_2), \\
\text{subject to} & \quad \pi c_1 \leq (1 - I) + Lp, \\
& \quad (1 - \pi)c_2 \leq (I - L)R - e_2, \\
& \quad e_2 \geq \rho e_0, \\
& \quad R \geq \gamma p, \\
& \quad pL \leq A, \\
& \quad I \leq 1 + e_0, \\
& \quad L \leq I.
\end{align*}
\]

The expected utility of a consumer is specified in Equation (2). The budget constraints for periods one and two are given by Equations (3) and (4). Investors may transfer \( L_p \)
to consumers in $t = 1$ in exchange for $L$ units in $t = 2$. Later this will referred to as wholesale funding or asset sales. Equation (5) denotes the participation constraint of the intermediary, and Equation (6) the participation constraint of investors. Equation (7) denotes the resource constraint on investors capital $A$ ("liquidity") in the interim period. Finally, Equations (8) and (9) denote the constraint on initial investment as well as the constraint on the units of assets that can be sold in the interim period.

Notice first that Equation (9) cannot be binding. Because of the Inada conditions, we require that $c_2 > 0$, thus $L < I$. It is obvious that the two budget constraints are always binding. Furthermore, the participation constraint of intermediaries must also be binding. If we had $e_2 > e_0 \rho$, we could lower $e_2$ to $e_0 \rho$, thereby relaxing the second-period budget constraint. We therefore have that $e_2 = e_0 \rho$. However, $e_0 > 0$ cannot be optimal. Because $\rho > R$, we can reduce $e_0$ and thereby relax the second period constraint. It therefore follows that $e_0 = e_2 = 0$. Because the required return on equity is higher than the asset return, no equity is used for intermediation in the first best. As we will see below, if we also consider incentive-compatibility, it may become necessary to force the intermediary to inject some capital.

Let us now turn towards the use of interim liquidity, i.e., the sale of assets in period 1. In the first-best, it also has to hold that the participation constraint of investors is binding. Whenever $p < R/\gamma$ and $pL < A$, we can increase $p$ and thereby relax the period 1 constraint. Whenever $p < R/\gamma$ and $pL = A$, we can increase $p$ and decrease $L$ as much as necessary, thereby relaxing the period 2 constraint. Therefore, it holds that $p = R/\gamma$.

We are now left with a maximization problem with two weak inequalities.

$$\max_{(c_1, c_2, I, L) \in \mathbb{R}_+^4} \pi u(c_1) + (1 - \pi) u(c_2),$$

subject to

$$\pi c_1 = (1 - I) + LR/\gamma,$$

$$\pi c_2 = (I - L)R,$$

$$LR/\gamma \leq A,$$

$$I \leq 1.$$  

Depending on the model parameters $A, R, \gamma$ and $\pi$, as well as on the shape of the utility function, the first-best program now has three solution candidates which depend on the available interim outside liquidity $A$. As discussed in detail in the appendix, investment in storage is only optimal if $A$ is small and becomes unnecessary when $A$ is sufficiently large.

For the remaining part of the paper, we will assume that we are in the case in which
the outside liquidity $A$ is always large enough such that the arbitrage budget constraint Equation (13) is not binding, which also implies that storage is not used, i.e., $I^* = 1$. This property is satisfied by the following assumption:

**Assumption 1.** $A \geq \xi \equiv \gamma - \frac{1}{\eta} \frac{\pi R}{(1-\pi) + \pi \gamma^{1-\frac{1}{\eta}}}$. 

For a detailed discussion of the implications of Assumption 1, see Appendix A, in which we discuss the characterization of the first-best in case the assumption is not satisfied.

Assumption 1 restricts our attention to the case that is most simple in terms of notation compared to the cases discussed in the Appendix. This allows us to focus on a setup where intermediation optimally rely exclusively on investors providing liquidity and refrain from investing in storage, i.e., it relies on a financing strategy that is comparable to wholesale funding: Instead of relying on stored goods, it is optimal to rely on outsiders to provide funds in exchange for claims on future returns when liquid funds are needed. However, as will become clear throughout the paper, our results will also hold for lower but positive level of $A$.

Given this assumption, we can derive the optimal allocation:

**Lemma 1 (First Best).** The first-best allocation is characterized by

$I^* = 1$ \quad $L^* = \xi \gamma / R$ and $e_0 = e_2 = 0,$

and optimal consumption is given by

$$c^*_1 = \gamma^{\frac{1}{\eta}} \frac{R}{(1-\pi) + \pi \gamma^{1-\frac{1}{\eta}}} \quad \text{and} \quad c^*_2 = \frac{R}{(1-\pi) + \pi \gamma^{1-\frac{1}{\eta}}}.$$ \hfill (15)

The optimal consumption profile is determined by the return of the long asset $R$, the required return of investors $\gamma$, and the consumers’ risk preferences $\eta$. Observe that because the endowment of investors $A$ is larger than the threshold $\xi$, it is optimal to exclusively invest in the productive technology and transfer some of the investment in exchange for investors’ endowment in the interim period, i.e., there is no storage. For the extreme case of $\gamma = 1$ and $A \geq \xi$, we are able to attain perfect risk sharing, the optimal consumption profile would be $(R, R)$. For $\gamma \in (1, R)$ and $A \geq \xi$, the optimal allocation is characterized by risk sharing between early and late consumers.

Diamond and Dybvig (1983) restrict attention to utility functions with a relative risk aversion strictly larger one. They show that under this assumption, risk sharing implies a transfer of patient to impatient consumers, i.e., they show that $c^{DD}_1 > 1$. However,
this condition also enables self-fulfilling runs. In our setup with constant relative risk and \( \eta > 1 \), we get a similar result with respect to risk-sharing, i.e., \( c_1^* > R/\gamma \) whenever \( \eta > 1 \). But as we shall see in next subsection, this condition also has similar implications for fragility and self-fulfilling runs.

As indicated above, see Lemma 4 in the Appendix for the first best if we do not restrict \( A \geq \xi \). For \( A < \xi \), the outside liquidity constraint (13) is binding, so it is optimal to use all of the available endowment of investors. Furthermore, there exists some threshold \( \xi_0 < \xi \) below which partial investment in storage is optimal. In the extreme case if \( \gamma = R \) or \( A = 0 \), the optimal allocation is equivalent to the optimal consumption profile in the Diamond and Dybvig model, which is nested in our model for CRRA utility.

3.2. Intermediary Implementation

Let us now turn towards the implementation of the first best allocation via a demand-deposit contract offered by intermediaries. We first have to analyze whether consumers will be willing to lend to intermediaries, given that intermediaries may invest in the shirking technology. We will show that this is possible, as the demand-deposit contracts allow depositors to discipline the intermediary. In a second step, we show that the disciplining element of the implementation is associated with financial fragility in the sense that panic-based runs may take place in the interim period.

Disciplining Demand-Deposit Contracts

We assumed that consumers cannot invest in the technologies directly, but only via intermediaries. In this section, we consider the agency problems that are associated with intermediaries offering demand-deposit contracts. To this end, let us first devote more attention to the timing and the action space of consumers and intermediaries.

A consumer can choose whether and where to deposit his endowment in period 0, and an intermediary can then choose how to invest this endowment. In period 1, consumers learn their type and observe the intermediary’s investment choice from the initial period, and they can decide whether to withdraw based on this information.

Let us assume that competition among intermediaries forces them to offer the first-best demand-deposit contract \((c_1^*, c_2^*)\) in exchange for the consumers endowment. In period 1, consumers have the possibility to withdraw the promised amount of \( c_1^* \), or to wait until period 2. We have assumed that an intermediaries investment decision \( I_{shirk} \) is

\[ \text{Notice that in case of unsecured wholesale funding, one would have to worry about the behavior of investors as well, compare Luck and Schempp (2014). However, for the case of asset sales or collateralized lending we do not have to worry about investors as long as they do not collude in order to extract rents from consumers.} \]
not observable in $t = 0$, but becomes publicly observable before consumers make their withdrawal decision in $t = 1$.

Consider the following consumer strategy for the period-1 subgame: She withdraws if she turns out to be impatient or if the intermediary has chosen $I_{\text{shirk}} > 0$, and she does not withdraw if she turns out to be patient and the intermediary has chosen $I_{\text{shirk}} = 0$. If all consumers follow this strategy, this strategy profile constitutes a Nash Equilibrium in the period-1 subgame for any investment decision of the intermediary.

Assume the intermediary has only invested in the shirking technology, i.e., $I_{\text{shirk}} = 1$. It is immediately clear that she cannot receive more than $R_{\text{shirk}} < 1$ for her one unit of the shirking technology because the private benefit $B$ is assumed to be non-transferable. Given that $c^*_1 > R/\gamma > R_{\text{shirk}}$, the intermediary will have to sell all its funds to serve the withdrawing depositors and will have no funds left in the final period. Therefore, it is indeed optimal to withdraw given $I_{\text{shirk}} = 1$ and all other agents withdraw.

Now assume the intermediary has only invested in the productive technology, i.e., $I = 1$ and $I_{\text{shirk}} = 0$. Given that only impatient consumers withdraw, the intermediary will be able to serve all early consumers by selling $L^*$ units of her investment to investors. As we assumed that $A \geq \xi = \pi c^*_1$, the investors’ funds are sufficient to payout all early depositor. As $c^*_2 > c^*_1$, it is optimal for patient consumers to wait.

This withdrawal strategy is a credible punishment strategy, and it uses the threat of a bank run as a disciplining device: Because the intermediary knows that she will experience a run in $t = 1$ whenever she invests in the shirking technology, she will experience a bank run and will therefore not be able to enjoy the private benefit $B$. Therefore, she does not invest in the shirking technology in the first place. This disciplining effect of short-term debt is reminiscent of the findings of Calomiris and Kahn (1991), and Diamond and Rajan (2001), and allows intermediaries to implement the first-best allocation via demand-deposit contracts:

**Proposition 1** (Implementation of the First Best). *There exists a subgame-perfect Nash Equilibrium in which the first best $(c^*_1, c^*_2)$ is implemented by the intermediaries using demand-deposit contracts.*

Note that there also exists a continuum of equilibria in which the bank chooses to partially invest in the shirking technology, but is not disciplined by the depositors. We discuss these in the Appendix [4].

12
3.3. Fragility

While short-term debt is disciplining in our model, it also is a source of fragility. In fact, the model exhibits multiple equilibria in the period-1 subgame. However, depending on the amount of investors’ funds $A$, the run equilibria are qualitatively different. As long as the amount of funds $A$ is sufficiently large, potential runs on some intermediaries do not affect other intermediaries. However, if the endowment of investors $A$ is relatively small, liquidity can become scarce in case of a run on many intermediaries. This puts the market for liquidity under stress and deteriorates the funding conditions of other intermediaries.

The price $p$ of assets sold in period 1 depends on the aggregate amount $L$ of assets sold as soon as the investors’ resource constraint becomes binding. As long as the resource constraint is not binding, competition among investors ensures that the price is equal to the investors’ willingness to pay. Thus, if $A$ is so large such that $L$ units of the asset can purchased by investors at price $p = R/\gamma$, this is the market-clearing price, i.e., the price is equal to the assets rate of return $R$ divided by the rate of the investors’ outside option $\gamma$. If, however, $A$ is scarce relative to the amount $L$ of assets sold (i.e., if $A$ is not sufficient to purchase $L$ units at price $R/\gamma$), the market clears via cash-in-the-market pricing, i.e., it must hold that $pL = A$.

Given $L$, the amount of assets sold, the price of the assets in period 1 is given by

$$p(L) = \begin{cases} 
\frac{\gamma}{R} & \text{if } A\frac{\gamma}{R} \geq L \\
\frac{A}{L} & \text{if } A\frac{R}{\gamma} < L.
\end{cases} \quad (16)$$

Recall that by Assumption[1] we restrict our attention to the case in which intermediaries exclusively invest in the productive technology, i.e., $I^* = 1$, so the amount of assets sold is at most one. The price for assets in period 1 is depicted in Figure[1] for the case that $A < R/\gamma$.

As long as $A \geq R/\gamma$, liquidity cannot become scarce, and the price is always given by $R/\gamma$. However, if $A < R/\gamma$, runs on some intermediaries can have negative external effects on others. If sufficiently many intermediaries experiencing a run, the price $p$ gets depressed, thus deteriorating the refinancing condition of other intermediaries.

**Micro-Fragility: Runs on Single Institutions**

Let us start considering the stability of a single intermediary. Notice that on the one hand, the price on the secondary market is limited by the investors’ willingness to pay, i.e., $p \leq R/\gamma$, but on the other hand, the optimal demand-deposit contract promises an early consumption level that is strictly larger than this amount, $c_1^* > R/\gamma$. Because $p <
Figure 1: This graph depicts the potential fire-sale price for the case $R/\gamma > A$. In this case, a run may lead to depressed fire-sale prices.

$c_i^*, it holds true that if all depositors of one specific intermediary $i$ run, this intermediary has to sell all assets, but still cannot fulfill all its obligations to her depositors. This particular intermediary becomes illiquid and insolvent already in period 1, and in particular could not serve any late consumer. Thus, a run on intermediary $i$ constitutes an equilibrium.

**Lemma 2 (Individual Runs).** Assume that intermediaries choose the first-best investment level and demand-deposit contract. There exists a Nash Equilibrium in the period-1 subgame in which there is a run on some intermediary $i$, inducing a complete asset sale and immediate illiquidity and insolvency of this intermediary. In particular, there exists an equilibrium in which there is a run on all intermediaries.

Notice that the run on a mass $j$ of intermediaries does not necessarily affect the remaining mass $1 - j$ of other intermediary. If there is sufficient investor capital $A$, the price on the market remains high enough to make it possible that there exists an equilibrium where some mass $j$ of intermediaries face a run, but rest does not does not. The reason is if $A$ is large enough and if (conditional on $A$) the mass $j$ of intermediaries in stress is sufficiently low, the price in the secondary market is high enough to make “prudent” behavior at the intermediaries $1 - j$ compatible in equilibrium with runs elsewhere. Nonetheless, it may be true that all intermediaries are experiencing a run at the same time.
Macro-Fragility: Systemic Runs and Cash-in-the-market-pricing

Notice first that if \( A > \frac{R}{\gamma} \), it holds that \( p(L) = \frac{R}{\gamma} \) for all \( L \). This means that even in case of an economy-wide run, the price on the secondary market is unaffected and there is no binding cash-in-the-market constraint. This also implies that if all intermediaries except for \( i \) had a run, this run would not affect \( i \) at all, because it can sell the designated amount \( L^* \) at the expected price \( p = \frac{R}{\gamma} \), so it could refinance at the ex-ante expected conditions.

Now consider the case of \( A < \frac{R}{\gamma} \). This implies that there is cash-in-the-market pricing in case of an economy-wide run, implying that \( p(1) = A \). This implies that if all intermediaries but one are experiencing a run, the intermediary that is not experiencing a run will yet face deteriorated funding conditions. We refer to runs as “systemic runs” if they induce cash-in-the-market pricing and thus affect the overall funding conditions.

**Proposition 2** (Systemic runs). Assume that \( A < \frac{R}{\gamma} \), and assume that intermediaries choose the first-best investment level and demand-deposit contract. Then there exist “systemic runs”, i.e., an economy-wide run in the period-1 subgame leads to cash-in-the-market pricing and thus a deterioration of overall funding conditions.

Proposition 2 shows that the ability to withdraw early induces diligent behavior of the intermediary, so short-term debt has a disciplining effect in our model. However, there always exists multiple equilibria. In one class of equilibria, only some single institutions experience runs while others are not, and the latter ones remain completely unaffected. From Lemma 2 we learn that runs are always possible (on single institutions but also an economy-wide runs). However, the runs on single institutions are independent from each other. From Proposition 2 we learn that runs are contagious via deteriorated funding conditions only if \( A < \frac{R}{\gamma} \), i.e., if investor capital is scarce. Whenever \( A < \frac{R}{\gamma} \), there exists a second class of equilibria, in which runs also become contagious in the sense that they affect funding conditions of other institutions. The second type of run will be particularly important when we analyze later, how runs in the shadow banking sector may affect funding conditions for the regulated banking sector.

3.4. Financial Markets Implementation

Until this point, we have by assumption ignored the possibility of implementing an allocation via a financial market instead via intermediaries. The allocation that can be attained via a financial market in which consumers invest in the technologies directly and trade with investors in \( t = 1 \), is \((c_1^{fm}, c_2^{fm}) = (\frac{R}{\gamma}, R)\). This allocation, however, only coincides with the first best if \( \eta \to 1 \), i.e., if \( u(c) = \ln(c) \). This reminiscent of the
Nonetheless, we need to make the investment restriction: If we allowed for the coexistence of financial markets intermediaries, the incentive to conduct side trading would destroy the ability to implement the first best via intermediaries, due to the same reasoning as in Jacklin (1987) and Farhi et al. (2009). If intermediaries offered the first-best demand-deposit contract, a consumer has an incentive to invest his endowment in the productive technology and consume the returns $R > c^*_2$ if he turns out to be patient, and to trade with a patient depositor otherwise, thereby consuming $c^*_1$ units.

4. Deposit Insurance and Optimal Bank Regulation

As we have seen in the previous section, the first best is implementable through non-regulated intermediaries, but this implementation is fragile in the sense that there always exist run equilibria in the period-1 subgame. To eliminate such panic based bank runs, we consider a deposit insurance.

In a setup without aggregate uncertainty and with multiple equilibria, introducing a deposit insurance that is credible may eliminate the adverse run equilibrium at no cost, e.g. as in Diamond and Dybvig (1983). By guaranteeing patient consumers to get at least as much in the final period than in the interim period, the strategic complementarity is destroyed. Thus, the deposit insurance is never tested in equilibrium and thus costless.

In our setup, however, a deposit insurance – if implemented without further regulatory policy measures – can give rise to opportunistic behavior on the part of intermediaries which imposes high costs the provider of such deposit insurance. The reason is that in the presence of deposit insurance, consumers do not care about the investment behavior of the intermediary, thus eliminating the disciplining effect of short-term debt. Even if they know that the intermediary will be insolvent in the second period, they do not run because they know that the deposit insurance will give them the amount that there demand deposit contract entitles them to. Therefore, an intermediary has incentives to invest in the shirking technology, unless she has a sufficiently high equity stake in her business.

Given the moral hazard problem arising from the deposit insurance, there exists an optimal regulatory response. In the first best, the intermediary does not have any skin in

---

9An alternative measure is to allow intermediaries to suspend convertibility. We abstract from this policy measure. One can, however, easily see that the discussion below would be equivalent under suspension of convertibility: suspension of convertibility may successfully prevent panic based runs but also undermines the disciplining effect of demand deposit contracts. If banks are able to suspend convertibility, regulation is required to ensure diligent behavior of intermediaries.
the game, and the participation constraint $e_2 \geq \rho e_0$ is trivially satisfied by $e_0 = e_2 = 0$. This is efficient because there is no need to provide the intermediary with incentives, and given $\rho > R$, it would be expensive to use the intermediary’s funds.

If a regulator wants to rule out moral hazard he can do so by forcing the intermediary to hold a sufficient equity stake in her intermediation business. This is a classic skin-in-the-game argument. To insure diligence, the incentive compatibility constraint of the intermediary has to be satisfied. It is given by

$$e_2 \geq (1 + e_0)B. \quad (17)$$

At the same time, the intermediary’s participation constraint, $e_2 \geq \rho e_0$, still needs to be fulfilled.

In the second best, both constraints are binding, i.e., $e_2 = (1 + e_0)B$ and $e_2 = \rho e_0$, yielding the second-best equity stakes

$$e_{0*} = \frac{B}{\rho - B}, \quad \text{and}$$

$$e_{2*} = \frac{\rho B}{\rho - B}. \quad (18)$$

Recall that we assumed that it is costly to use intermediary equity. Therefore, in an optimal regulatory regime that tries to prevent the intermediary to invest in the shirking technology, as little as possible intermediary capital is used, but enough to ensure diligent behavior. Given this second-best equity level, we can derive the second best demand-deposit contract.

**Proposition 3 (Second-Best Contract).** Assume that demand deposits are protected by a credible deposit insurance. Optimal bank regulation requires intermediaries to satisfy an equity-to-debt ratio of $B/(\rho - B)$, and intermediaries will hold exactly $e_0 = B/(\rho - B)$. There exist no run equilibria in the period-1 subgame. Given that $\xi \leq A$, investment and sales are given by

$$I^{**} = 1 + \frac{B}{\rho - B} \quad \text{and} \quad L^{**} = \pi \gamma^{-\frac{1}{\eta}} \frac{R - \frac{\rho - R}{\rho - B} B}{(1 - \pi) + \pi \gamma^{1 - \frac{1}{\eta}}}, \quad (20)$$

and the optimal consumption is given by

$$c_{1*}^{**} = \gamma^{-\frac{1}{\eta}} \frac{R - \frac{\rho - R}{\rho - B} B}{(1 - \pi) + \pi \gamma^{1 - \frac{1}{\eta}}} \quad \text{and} \quad c_{2*}^{**} = \frac{R - \frac{\rho - R}{\rho - B} B}{(1 - \pi) + \pi \gamma^{1 - \frac{1}{\eta}}} \quad (21)$$
In the regime with a deposit insurance, the consumption levels are decreasing in the private benefit $B$ as well as in the required return of intermediaries $\rho$. Obviously, first best (Lemma 1) and second best coincide if $B = 0$ or $\rho = R$. For any other $B > 0$ and $\rho > R$, the second-best consumption levels are strictly lower. In fact, the case of $B > 0$ but $\rho = R$ is very interesting. In this case, using intermediary capital is not costly, the first-best can always be implemented by using intermediary capital and investing it in the production technology until incentives are provided.

Importantly, there are no run equilibria in the interim period. The allocative inefficiency comes with the benefit of financial stability. However, as we will emphasize in the next section, this overall stability can only be attained if we exclude the possibility of regulatory arbitrage.

5. Regulatory Arbitrage and Fragility

In the previous section, we have ignored the possibility of regulatory arbitrage. In the following we assume that the regulator provides a deposit insurance and regulates those intermediaries that are covered by the deposit insurance, hence referred to “commercial banks”. However, we assume that it is also possible for intermediaries place themselves outside of the regulatory perimeter of banking. Intermediaries that engage in this kind of regulatory arbitrage are referred to as “shadow banks” in the following. In this case, they will neither be regulated nor covered the deposit insurance. However, shadow banks are disciplined in their investment behavior by demand-deposit contracts.

In the following, we want to analyze a situation in which both regulated commercial banks and shadow banks exist, and analyze how systemic risk that emerges in the shadow banking sector can spread to the commercial banking sector. In our model, the coexistence of regulated banking and shadow banking results form an additional assumption concerning the heterogeneity of the depositors with respect to their “taste for shadow banking”, which we model as a cost of moving to a shadow bank.

In period 0, consumers can choose whether to deposit their endowment in a regulated bank or in a shadow bank. We assume that consumers do not care about potential self-fulfilling runs on shadow banks as runs are considered to be zero probability events. However, depositing at a shadow bank is assumed to come at some opportunity cost. We assume that investors are initially located at a regulated bank. For some consumer

\[10\] While by legal standards shadow banks do not offer demand deposits in reality, they do issue claims that are essentially equivalent to demand deposits, such as equity shares with a stable net assets value (stable NAV). For tractability, we will assume that shadow banks are literally taking demand deposits.
i, switching to a shadow bank comes at a cost of $s_i \in \mathbb{R}^+$, where $s_i$ is independently and identically distributed according to the distribution function $G$. We assume that $G$ is a continuous function that is strictly increasing on its support $\mathbb{R}^+$, and that $G(0) = 0$. The switching cost is assumed to enter into the investors’ utility additively separable from the consumption utility.

This switching cost should not be taken literally. A possible interpretation is the forgone service benefits that depositors lose when leaving commercial banks, such as payment services and ATMs. These are likely to be heterogeneous among the population of depositors. E.g., while consumers may desire to be at a bank that provides payment services, a firm may have less benefits from such and may be more likely to switch.

**Coexistence of Banks and Shadow Banks**

First of all, consider the contracts that can be offered. A regulated bank will offer a contract as discussed in Proposition 3 so the consumption levels are given by $(c_{b1}, c_{b2}) = (c_{1}^{**}, c_{2}^{**})$, where the superscript $b$ stands for bank. The expected utility of a bank customer is decreasing in $B$ and $\rho$. A shadow bank will offer the unconstrained contract described in Lemma 1, so the consumption profile is given by $(c_{sb1}, c_{sb2}) = (c_{1}^{*}, c_{2}^{*})$, where the superscript $sb$ stands for shadow bank.

When born, a consumer can decide whether he wants to remain at a bank, or pay the switching costs and get the consumption profile of the shadow banking contract. Thus, a consumer decides to become a customer of the shadow bank if

$$U(c_{sb1}, c_{sb2}) - s_i \geq U(c_{b1}, c_{b2}).$$

Let us define $s^*(B, \rho)$ such that a consumers with switching cost $s_i = s^*$ is indifferent between the two sectors, i.e.,

$$s^* = U(c_{1}^{sb}, c_{2}^{sb}) - U(c_{1}^{b}, c_{2}^{b}).$$

(22)

**Lemma 3** (Size of Shadow Banking). The size of the shadow banking sector is given by $G(s^*)$. The cost threshold $s^*$ that makes consumers indifferent, and thus also sector size $G^*$, are increasing in $B$ and $\rho$. It holds that $G^* = 0$ if $B = 0$ or $\rho = R$.

The relative size of the two sectors is determined by the distribution $G$ as well as the two parameters $B$ and $\rho$. The size of the shadow banking sector increases with the threat of moral hazard (i.e., in the intermediaries’ private benefits from shirking $B$) and in the cost of intermediary equity $\rho$.

**Systemic Runs on Shadow Banks and Contagion**

In this paragraph, we derive the conditions under which a runs in the shadow banking sector are systemic, and we will show that not only systemic runs on shadow banks affect
regulated banks.

Recall that because \( c_1^* \geq R/\gamma \), and a run on one individual shadow bank constitutes an equilibrium independent of the amount of arbitrage capital \( A \), because the claims of all consumers withdrawing early cannot be satisfied via complete liquidation, compare Lemma 2. Moreover, a run on all shadow banks also constitutes an equilibrium for the same reason. Such a run on all shadow banks is not necessarily contagious. However, if the shadow banking sector is sufficiently large, or the amount of arbitrage capital too small, then the run on shadow banks soaks up so much liquidity it deteriorates the funding conditions for commercial banks.

We will now analyze under which condition a run on shadow banks leads to cash-in-the-market pricing. In case of a run on the shadow banking sector, all shadow banks try to serve all withdrawing depositors. In order to fulfill their obligations, they sell all their assets, i.e., the shadow banking sector sells a total amount of \( G(s^*) \) units. As long as there is no cash-in-the-market pricing, shadow banks thus absorb an amount of liquidity \( G(s^*)R/\gamma \). However, this does not suffice to serve all customers because \( c_1^* > p \geq R/\gamma \). In addition, commercial banks also need an amount of \( [1 - G(s^*)]\pi c_1^{**} \) of liquidity to satisfy their withdrawing impatient consumers. A run on shadow banks is not compatible with a price \( p = R/\gamma \) (i.e., it leads to cash-in-the-market pricing) and thus has a negative effect on regulated banks if the sum of these two terms exceeds the available funds \( A \).

**Proposition 4** (Coexistence and Runs on Shadow Banks). When regulated banks and shadow banks coexist, a run on all shadow banks leads to cash-in-the-market pricing whenever the shadow banking sector is large relative to the budget of investors, specifically, if

\[
G(s^*) > \frac{A - \pi c_1^{**}}{R/\gamma - \pi c_1^{**}} = \bar{g}(A).
\]  

In this case, the run also affects commercial banks because their funding conditions deteriorate. There exists some threshold \( \bar{g}(A) > \bar{g}(A) \) such that in case of a run, regulated banks become insolvent if \( G(s^*) > \bar{g}(A) \).

If \( A < R/\gamma \), it holds that \( \bar{g}(A) < 1 \), i.e., the price in a run decreases whenever the shadow banking sector is large enough. Although regulated banks cannot experience runs because they are covered by the deposit insurance, they suffer from deteriorated funding conditions via cash-in-the-market pricing that are caused by the runs on shadow banks. That is, in case of a run on all shadow banks, the asset price in the secondary market will be lower than initially expected. Thus, commercial banks will have to refinance themselves at worse conditions than initially expected. However, banks’ equity cushion
may be able to absorb these losses. Given a relatively large shadow banking sector, the fire-sale price will be so low such that banks become insolvent in \( t = 2 \), or already illiquid and insolvent in \( t = 1 \). Importantly, this kind of contagion does not stem from direct contractual linkages between the two sectors such as explicit or implicit liquidity guarantees.

![Figure 2: This graph depicts the fire-sale price in case of a run on all shadow banks. A run leads to cash-in-the-market pricing and thus affects regulated banks if \( G(s^*) > \bar{g}(A) \).](image)

In the last section we have shown that if there is deposit insurance and regulation, but regulatory arbitrage is not possible, it holds that the economy attains an allocation which is inefficient compared to the first best, but exhibits no fragility. If we assume that regulatory arbitrage is possible, this may no longer be true. Whenever the shadow banking sector is relatively large to the available capital of investors, runs on shadow banks become possible and can affect regulated banks via deteriorated funding conditions.

### 6. Effects of Wholesale Funding Restrictions

A natural question that arises in this context is to ask whether limiting wholesale funding of commercial banks can improve financial stability. In fact, some regulatory reforms propose such restrictions on wholesale funding. In the following, we analyze the most simple and most extreme case: a complete prohibition of wholesale funding for regulated banks, i.e., the restriction that \( L = 0 \). We show that this shuts down the contagion channel, but the associated allocation is less efficient.

\[ \text{E.g., the Liikanen report speaks of a mandatory separation of banking business in deposit bank and trading entity, where a 'deposit bank' is exclusively funded by insured deposits.} \]
In the case of a restriction on wholesale funding, commercial banks can only offer an allocation where liquidity is provided by investing in storage. The optimal allocation can be calculated by solving the following problem:

\[
\max_{(c_1,c_2,I) \in \mathbb{R}^3_+} \pi u(c_1) + (1 - \pi)u(c_2),
\]

subject to

\[
\pi c_1 = (1 - I)
\]

\[
(1 - \pi)c_2 = IR - e_2
\]

\[
I \leq 1 + e_0
\]

\[
e_2 \geq (1 + e_0)B
\]

\[
e_2 \geq pe_0
\]

The optimal allocation is thus given by:

\[
c_1^r = \frac{R^{1-\frac{1}{\pi}} - \frac{e_2 R}{\rho B} B}{\pi R^{1-\frac{1}{\pi}} + (1 - \pi)}
\]

\[
c_2^r = \frac{R - \frac{e_2 R}{\rho B} B}{\pi R^{1-\frac{1}{\pi}} + (1 - \pi)}
\]

Note that this optimal contract under the exclusion of wholesale funding \((c_1^r, c_2^r)\) is identical to the first-best allocation of the Diamond and Dybvig model whenever \(B = 0\) or \(\rho = R\).

Shadow banks still offer the same contract as above, i.e., \((c_{1b}, c_{2b}) = (c_1^r, c_2^r)\). Given the switching cost \(s_i\), let us define \(s'(B, \rho, L = 0)\) such that a consumer with switching costs \(s_i = s'\) is indifferent between the two sectors, i.e.,

\[
s' = U(c_{1b}, c_{2b}) - U(c_1^r, c_2^r).
\]

One can directly see that the restrictions will shield regulated banks from adverse consequence of a run in the shadow banking sector, but will at the same time lead to a larger shadow banking sector in equilibrium. That is, we will find that \(s' > s^*\).

**Proposition 5.** Wholesale funding restrictions successfully shield regulated banks from the adverse consequences of runs in the shadow banking sector. However, the shadow banking sector will be larger, \(G(s') > G(s^*)\).

Wholesale funding restrictions thus eliminate the fragility together with a deposit insurance altogether. They do, however, induce a further allocative inefficiency by deteriorating the consumption profile of bank customers and by pushing more depositors in the shadow banking sector.
7. Liquidity Guarantees

So far, we have restricted attention to intermediaries becoming either regulated banks or unregulated shadow banks. An interesting question is how our results change when banks and shadow banks are interdependent not only via effects on secondary markets, but if they are run by the same intermediary. We will thus analyze the effect of direct contractual linkages.

Private Optimality of Liquidity Guarantees

If an intermediary operates a bank and a shadow bank, she has an incentive to support her shadow bank in case of a run. Recall from Lemma 2 that there may exist runs on single institutions that are not inducing cash-in-the-market pricing. These types of runs can be eliminated if a regulated branch of an intermediary provides a liquidity guarantee for its unregulated arm. From the perspective of an individual institution, it is thus optimal to provide this support guarantee.

Systemic Runs

While it is optimal from the perspective of a single institution to provide a liquidity guarantee, it leads to an increased parameter space for runs on the aggregate level. In fact, runs will take place as if there was no deposit insurance. However, the existing regulation will induce and allocating inefficiency.

Proposition 6. Assume that intermediaries can own a regulated bank and a shadow bank at the same time. It is privately optimal for each intermediary to guarantee the liquidity for her shadow bank branch by using funds from her regulated bank. In turn, this decreases the threshold size of the shadow banking: A systemic run now already occur and affect regulated banks if

\[ G(s^*) \geq \frac{A - c_1^*}{c_1^* - c_2^*} < \bar{g}(A). \]

This shows that there is a clear benefit of preventing direct contractual linkages via regulation. However, as shown in the earlier section, it may not be sufficient.

8. Discussion

This papers provides a banking model in the tradition of Diamond and Dybvig (1983) in which banks optimally rely on wholesale funding next to standard deposit contracts. We show that we can either have the disciplining effect of short-term debt which is

\(^{12}\)See discussion on sponsor support Segura (2014).
accompanied by panic-based runs, or we can have a safety net and regulation which eliminate runs, but induce the emergence of a fragile shadow banking through regulatory arbitrage. We then show that regulatory arbitrage endangers financial stability via deteriorated funding conditions that result from runs on shadow banks.

We suggest two take-aways from our analysis: The ideal policy measure would prevent the circumvention of regulation. This would allow to attain a constrained efficient allocation in which financial stability is ensured by deposit insurance, and diligence is ensured through capital requirements. If, however, regulatory arbitrage cannot be directly prevented, attaining stability becomes difficult. One can consider to ring-fence the part of the banking sector that is covered by deposit insurance. However, besides prohibiting direct contractual linkages, one would then have to restrict or prohibit wholesale funding altogether. This would further increase the incentives to circumvent regulation and increase the unregulated shadow banking sector. Whether this is desirable remains an open question. Our analysis indicated that there is a trade-off between financial stability and efficiency.
Appendix A  First Best

As discussed in the text, we are now left with a maximization problem with two weak inequalities. We will now argue that depending on the model parameters, the first-best program now has three solution candidates.

\[
\max_{(c_1,c_2,I,L) \in \mathbb{R}_+^4} \pi u(c_1) + (1 - \pi)u(c_2),
\]

subject to

\[
\begin{align*}
\pi c_1 &= (1 - I) + LR/\gamma, \\
(1 - \pi)c_2 &= (I - L)R, \\
LR/\gamma &\leq A, \\
I &\leq 1.
\end{align*}
\]

Depending on the model parameters \(A, R, \gamma\) and \(\pi\), as well as on the shape of the utility function, the first-best program now has three solution candidates. In the first case \((A \geq \xi)\), it holds that \(I^* = 1\) and \(L^* < 1\), and the optimal allocation is characterized by

\[
u'(c_1) = \gamma u'(c_2)
\]

\[
\pi c_1 \gamma + (1 - \pi)c_2 = R.
\]

In the second case \((\xi_0 \leq A \leq \xi)\), we have that

\[
I^* = 1 \text{ and } L^* = A\gamma/R
\]

and optimal consumption is given by

\[
c_1^* = \frac{A}{\pi} \text{ and } c_2^* = \frac{R - A\gamma}{(1 - \pi)}.
\]

In the third case \((A < \xi_0)\), we have \(L^* = A\gamma/R\), and \(I^* < 1\), and the optimal allocation is characterized by

\[
u'(c_1) = Ru'(c_2)
\]

\[
\pi c_1 R + (1 - \pi)c_2 = R + (R - \gamma)A.
\]

\(\xi\) and \(\xi_0\) are defined as:

\[
\xi \equiv \gamma^{-\frac{1}{\eta}} \frac{\pi R}{(1 - \pi) + \pi \gamma^{1 - \frac{1}{\eta}}} < R/\gamma,
\]

and

\[
\xi_0 \equiv \frac{\pi R^{1 - \frac{1}{\eta}}}{(1 - \pi) + \pi \gamma^{\frac{1}{\eta}}} < \xi,
\]

25
Lemma 4 (First Best). If $A \geq \xi$, then the first-best allocation is characterized by

$$I^* = 1 \text{ and } L^* = \xi \gamma / R$$

and optimal consumption is given by

$$c^*_1 = \gamma^{-\frac{1}{\eta}} \frac{R}{(1 - \pi) + \pi \gamma^{1 - \frac{1}{\eta}}} \text{ and } c^*_2 = \frac{R}{(1 - \pi) + \pi \gamma^{1 - \frac{1}{\eta}}}.$$  \hspace{1cm} (44)

For $A \in (\xi_0, \xi)$, we have that

$$I^* = 1 \text{ and } L^* = A \gamma / R$$  \hspace{1cm} (45)

and optimal consumption is given by

$$c^*_1 = \frac{A}{\pi} \text{ and } c^*_2 = \frac{R - A \gamma}{(1 - \pi)}.$$  \hspace{1cm} (46)

Finally, if $A \leq \xi_0$, then the first-best allocation is characterized by

$$I^* = \frac{(1 - \pi)(1 + A) + \pi R^{-\frac{1}{\eta}} A \gamma}{(1 - \pi) + \pi R^{-\frac{1}{\eta}}} \text{ and } L^* = A \gamma / R,$$  \hspace{1cm} (47)

and optimal consumption is given by

$$c^*_1 = R^{-\frac{1}{\eta}} \frac{R + (R - \gamma) A}{(1 - \pi) + \pi R^{-\frac{1}{\eta}}} \text{ and } c^*_2 = \frac{R + (R - \gamma) A}{(1 - \pi) + \pi R^{-\frac{1}{\eta}}}.$$  \hspace{1cm} (48)

$\xi$ and $\xi_0$ are defined as:

$$\xi \equiv \gamma^{-\frac{1}{\eta}} \frac{\pi R}{(1 - \pi) + \pi \gamma^{1 - \frac{1}{\eta}}} < R / \gamma,$$ \hspace{1cm} (49)

and

$$\xi_0 \equiv \frac{\pi R^{1 - \frac{1}{\eta}}}{(1 - \pi) + \pi r^{\frac{1}{\eta}} \gamma} < \xi,$$ \hspace{1cm} (50)

### Appendix B Shirking Equilibria

Let us analyze the subgame in the interim period in more detail. If the intermediary has chosen $I_{\text{shirk}} = 1$, it is the dominant strategy of all consumers to withdraw their funds early, so this constitutes the unique Nash Equilibrium. If the intermediary has chosen some $I_{\text{shirk}} \geq 0$ small enough, “run” is not the dominant strategy. There still exist a
Nash Equilibrium in which patient consumers do not withdraw, but we are in a case of multiple equilibria in the period-1 subgame.

It is not trivial to argue that the threat of running is a disciplining device that rules out any shirking. There exist multiple equilibria in pure strategies for the whole game that differ in the extent of discipline that they ensure. There exists some $I_X \in (0,1)$ such that for any $I_x \in [0,I_X]$ there exists an equilibrium in which each consumer runs if and only if $I_{\text{Shirk}} > I_x$, and intermediaries choose $I_{\text{Shirk}} = I_x$. Because the intermediary enjoys the the benefit only when he does not experience a bank run in the interim period, there exists a set of interim-strategies of consumers that credibly threats to punish any shirking, so there exists a subgame-perfect equilibrium in which the intermediary does not shirk, but implements the first-best.

Small issue: Which incentives does the intermediary have? Why should behave, why should he offer the optimal contract, why should he intermediate at all? This requires him to have a minimal benefit from having a non-bankrupt bank in period 2, ideally increasing in the size of his business (perfect competition!).

References


