RIKSGÄLDS KONTORET
THE SWEDISH NATIONAL DEBT OFFICE

An analytical approximation of relative Cost-at-Risk

by Anders Holmlund

1. Introduction

The Debt Office has developed a simple model for arriving at an approximate figure for short-term relative Cost-at-Risk (RCaR) for Swedish central government debt. The method is not based on simulation but on analytical calculations.

The analytical RCaR measure shows how much higher than expected interest costs may be in a one-year perspective. In other words, it is a relative rather than an absolute measure. Expected interest costs are based on a situation where interest rates and exchange rates are unchanged, and the inflation rate is two per cent in keeping with the Riksbank's target.

In "Central Government Debt Management – Proposed Guidelines", the Debt Office describes the new measure and how it is influenced by the characteristics of the debt. In the proposal, there is also a discussion about how the new measure should be used in the management of government debt. This technical memorandum describes exactly how RCaR is calculated and what data are used in the analysis in the proposed guidelines.

2. The Debt Office model for approximating relative Cost-at-Risk

In principle, there are three risk factors for the cost of debt: the interest rate (all types of debt), the exchange rate (foreign currency debt) and inflation (inflation-linked debt).

If the *interest rate* climbs during a year, average interest rate on the debt rises by the increase in the interest rate multiplied by the portion of the debt whose interest rate is refixed. In this simple model, refixing of interest rates is assumed to be equal to the portion of the debt that falls due (also called yearly redemptions or maturities). Swedish and foreign interest rates are assumed to be perfectly correlated, while real interest rates are assumed to vary half as much as nominal ones.

If the *krona* weakens during a year, coupon payments on the foreign currency debt rise, measured in Swedish kronor. In addition, the Debt Office realises a larger (smaller) exchange loss (exchange gain) on the portion of the foreign currency debt that falls due.[1]

If *inflation* is higher than expected during the year, coupon payments on inflation-linked debt rise. In addition, more inflation compensation is realised on the portion of inflation-linked debt that falls due.

On the basis of how much falls due during the coming year, the percentages of inflation-linked and foreign currency debt in the total debt and the average coupon on the debt, we can calculate how much the costs increase for one unit of increase in each risk factor. With the help of historical, option-implied or assumed relationships between factors, we can then calculate confidence intervals for cost upturns. In other words, in this way we can arrive at an *analytic approximation* of RCaR.

On top of financial variables, we can add unexpected increases in the primary borrowing requirement. Such increases are assumed to be financed according to how the debt is structured from the beginning.

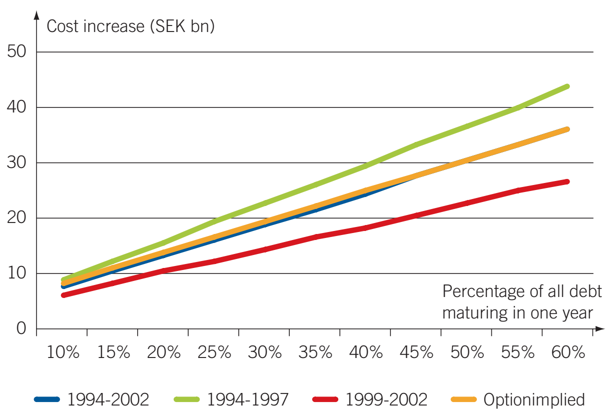
All equations are found in section 4.

3. Relative Cost-at-Risk as a function of maturity profile and percentage of foreign currency debt

The chart shows relative CaR as a function of the share of yearly redemptions, based on different historical periods, as well as data from the option market.

95% RCaR in a one-year perspective

(Foreign currency a constant 29% of total, inflation-linked 14%.
No shock in primary borrowing requirement)



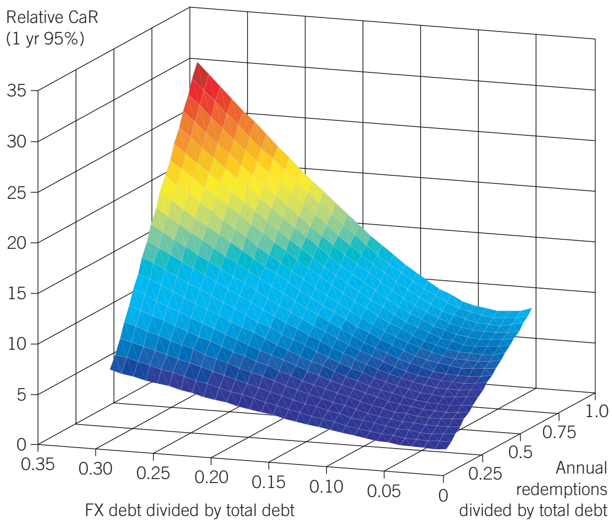
[1] By cost, we are referring here to economic costs. One difference compared to cash-basis costs is that exchange losses are considered realised when the loans fall due, regardless of whether the Debt Office refinances the loan or not. If the approach were instead cash-basis, we would look at the share that is repaid rather than the share that falls due.

RCaR increases linearly with the share of yearly redemptions. This is consistent with the conclusions in the Debt Office's report "Duration, Maturity Profile and the Risk of Increased Costs for Central Government Debt", which pointed out that the share of yearly redemptions is a better risk indicator than duration.

Depending on what historical period we use as the basis, RCaR rises by between SEK 2 and 4 billion for each five per cent climb in the maturity profile. The percentage of foreign currency debt in the total portfolio influences this figure – given a lower percentage of foreign currency debt, relative CaR rises more slowly if we increase the yearly maturity profile.

For this reason, it is interesting to view relative CaR as a three-dimensional function of the maturity profile and the percentage of foreign currency debt.

Relative Cost-at-Risk as a three-dimensional function



The chart is based on 12-month changes during the period 1998-2002

During this time period, CaR rises by SEK 2.3 billion for each 5 per cent increase in the maturity profile, if foreign currency debt represents 30 per cent of total debt. If the percentage of foreign currency debt is halved, CaR rises by SEK 1.2 billion for the same interval on the maturity profile.

The same pattern applies if we keep the percentage of foreign currency debt constant and vary the maturity profile. In other words, the percentage of foreign currency debt and the maturity profile have a mutually reinforcing effect on relative Cost-at-Risk.

4. Assumptions and equations for relative Cost-at-Risk [2]

Basic assumptions

The three risk factors – interest rate, exchange rate and inflation – are assumed to be normally distributed.

Swedish nominal debt and foreign currency debt are assumed to have the same share of yearly redemptions (25 per cent). Inflation-linked debt is assumed to have a smaller share of yearly redemptions (5 per cent).

The calculation concerns *the difference compared to ex-pected cost*, not the absolute cost level. The basic assumption is unchanged interest rates, unchanged exchange rates, two per cent inflation and a primary balance that turns out *as forecasted.*

Effect on cost of an interest rate increase

(Eq. 1)

$$\Delta c_r = \Delta r \times S(FP_{nom} \times (1 - w_{real}) + FP_{real} \times w_{real} \times 0.5)$$

where Δc_r = cost increase in SEK billion
Δr = change in interest rate
S = central government debt in SEK billion
FP = maturity profile
w = percentage (expressed as decimal)
nom = sub-index for nominal debt
$real$ = sub-index for inflation-linked debt

Please note: Swedish and foreign interest rates are assumed to move in tandem. Real interest rates are assumed to move half as much as nominal interest rates (which explains the 0.5 factor in the equation).

Effect on cost of a weaker krona

(Eq. 2)

$$\Delta c_{FX} = \Delta FX \times w_{FX} \times S \times (K_{FX} + FP_{nom})$$

where ΔFX = change in krona exchange rate (weakening)
K = coupon
FX = sub-index for foreign currency debt

Effect on cost if inflation is higher than expected

(Eq. 3)

$$\Delta c_{\pi} = (\pi - \pi^e) \times w_{real} \times S \times (K_{real} + FP_{real})$$

where π = actual inflation
π^e = expected inflation (in Sweden's case 2 per cent)

Borrowing requirement effects

The primary borrowing requirement may be larger than expected. It is assumed to be financed according to how the debt is structured. The interest rate on new loans is assumed to be the average coupon on the debt plus 1.96 standard deviations on the interest rate. [3]

[2] In the preparation of this memo, I encountered an error in the calculations. I have calculated a two-sided confidence interval instead of the one-sided interval I intended. This means that what we call 95 per cent RCaR is in fact 97.5 per cent RCaR. However, this error does not affect our qualitative conclusions in the Proposed Guidelines.

[3] The number of standard deviations must naturally be adapted to what confidence interval we have in the RCaR measure.

(Eq. 4)

$$\Delta c_{PSBR} = (K + 1.96\sigma_r)\Delta PSBR$$

where $K + 1.96\sigma_r$ = the interest rate at which the increased borrowing requirement will be financed

$\Delta PSBR$ = unexpected increase in the primary borrowing requirement

The primary borrowing requirement could be modelled in relation to the other variables, based on a historical correlation. But the primary borrowing requirement is influenced by many factors that this does not include, such as the GDP trend, divestment of state-owned companies and other political decisions. We have therefore chosen not to do this, but have instead made the primary borrowing requirement a static variable.

In principle, increased interest costs also lead to a larger borrowing requirement, which must be financed. It is, so to speak, a "cost on top of the cost increase". But this is a secondary effect that has been excluded in our calculations.

Overall risk

From equations (1), (2) and (3) we can calculate factor sensitivities for the three risk factors. Together with historical or option-implied standard deviations, we can calculate standard deviations for cost increases by risk factor.

Factor sensitivities are obtained by dividing equations (1), (2) and (3) by the change in each respective risk factor and multiplying with the standard deviation of each respective risk factor.

(Eq. 5)

$$\sigma_c^r = \frac{\Delta c_r}{\Delta r} \times \sigma_r$$

(Eq. 6)

$$\sigma_c^{FX} = \frac{c_{FX}}{FX} \times \sigma_{FX}$$

(Eq. 7)

$$\sigma_c^\pi = \frac{\Delta c_r}{(\pi - \pi^e)} \times \sigma_{(\pi - \pi^e)}$$

where, for example, σ_c^r means the standard deviation in cost increase based on interest rate change and σ_r is a standard deviation in the interest rate risk factor.

Since the correlation between the risk factors is not one, the total risk will be less than the sum of the individual risks. Overall risk is

(Eq. 8)

$$\sigma_C = \sigma^T \Omega \sigma$$

where Ω denotes the correlation matrix between the risk factors and σ is a column vector with standard deviations for each risk factor. If we add the (static) unexpected increase in the primary borrowing requirement, the equation becomes:

(Eq. 9)

$$\sigma_C = \sigma^T \Omega \sigma + \Delta c_{PSBR}$$

Examples of how relative Cost-at-Risk is calculated

At the end of July 2003, Swedish central government debt amounted to SEK 1,193 billion. The share of redemptions within one year is assumed to be 25 per cent for nominal krona debt and foreign currency debt, and 5 per cent for inflation-linked debt. Foreign currency debt represented 28.9 per cent of the total debt portfolio, and inflation-linked debt 13.7 per cent. The average coupon is assumed to be five per cent.

(Eq. 1)

$$\Delta c_r = \Delta r \times S \times \left(FP_{nom} \times (1 - w_{real}) + FP_{real} \times w_{real} \times 0.5\right)$$
$$\Delta c_r = \Delta r \times 1193 \times \left(0.25 \times (1 - 0.137) + 0.05 \times 0.12 \times 0.5\right)$$
$$\Delta c_r = \Delta r \times 261.0$$

(Eq. 2)

$$\Delta c_{FX} = \Delta FX \times w_{FX} \times S \times (K_{FX} + FP_{nom})$$
$$\Delta c_{FX} = \Delta FX \times 0.289 \times 1193 \times (0.05 + 0.25)$$
$$\Delta c_{FX} = \Delta FX \times 103.4$$

(Eq. 3)

$$\Delta c_\pi = (\pi - \pi^e) \times w_{real} \times S \times (K_{real} + FP_{real})$$
$$\Delta c_\pi = (\pi - \pi^e) \times 0.137 \times 1193 \times (0.05 + 0.05)$$
$$\Delta c_\pi = (\pi - \pi^e) \times 16.3$$

Historical data for the period 1994 – 2002 provide the following standard deviations (for twelve-month changes):

$$\begin{bmatrix} \sigma_r \\ \sigma_{FX} \\ \sigma_\pi \end{bmatrix} = \begin{bmatrix} 1.61\% \\ 6.42\% \\ 1.12\% \end{bmatrix}$$

This means that the vector with standard deviations for cost increases in the three factors is

$$\begin{bmatrix} \sigma_c^r \\ \sigma_c^{FX} \\ \sigma_c^\pi \end{bmatrix} = \begin{bmatrix} 1.61\% \times 261.0 \\ 6.42\% \times 103.4 \\ 1.12\% \times 16.3 \end{bmatrix} = \begin{bmatrix} 4.202 \\ 6.638 \\ 0.196 \end{bmatrix}$$

The correlation matrix for twelve-month changes in the same period is

$$\Omega = \begin{bmatrix} 1 & 0,11 & 0.55 \\ 0.11 & 1 & -0.02 \\ 0,55 & -0.02 & 1 \end{bmatrix}$$

The overall variance for unexpected cost increases will then be

(Eq. 8)

$$\begin{bmatrix} 4.202 & 6.638 & 0.196 \end{bmatrix} \begin{bmatrix} 1 & 0,11 & 0.55 \\ 0.11 & 1 & -0.02 \\ 0.55 & -0.02 & 1 \end{bmatrix} \begin{bmatrix} 4.202 \\ 6.638 \\ 0.196 \end{bmatrix} = 68.7$$

and the standard deviation is thus the square root of 68.7, which equals SEK 8.3 billion. This means that with 67 per cent certainty, the actual cost of the debt during next year will deviate by a maximum of SEK 8.3 billion from the forecast.

In Cost-at-Risk calculations, we are only interested in the risk that the cost will be larger than expected. We can then say that with 95 per cent probability, the cost will be no more than SEK 8.3 billion × 1.96 = SEK 16.3 billion higher than forecasted.

If the primary borrowing requirement meanwhile deteriorates by SEK 20 billion, the cost rises further:

(Eq. 4)

$$\Delta c_{PSBR} = (K + 1.96\,\sigma_r)\Delta PSBR$$
$$\Delta c_{PSBR} = (0.05 + 1.96 \times 0.0161) \times 20$$
$$\Delta c_{PSBR} = 1.63$$

A surprisingly large borrowing requirement – at the same time as interest rates, exchange rates and inflation show adverse changes – can thus result in an unexpected cost increase of around SEK 18 billion.

5. Data for the calculations

Historical calculations

Twelve-month Treasury bills are used as a measure of interest rate increase. This is a reasonable approximation, since a large proportion of borrowing each year is in Treasury bills and since a large proportion of the movements in the yield curve is explained by parallel shifts.

The change in exchange rate is measured as a change in a currency basket made up of 35 per cent USD and 65 per cent EUR. This is an approximation of the actual structure of the foreign currency debt. The dollar percentage is approximately equivalent to the sum of the USD, GBP and JPY percentages.

Inflation is measured as the twelve-month change in the Consumer Price Index (CPI).

Monthly data have been used, and all historical series have been obtained from the EcoWin database programme.

We use year-on-year changes in the monthly data. For example, this means that the period 1998 to 2002 contains underlying monthly data between January 1997 and December 2002.

Option-implied calculations

Implied volatility for one-year swaptions starting in one year has been used to approximate the risk of interest rate increase in a one-year perspective.

Implied volatility for foreign currency options on USD and EUR against SEK has been used to approximate the risk of a weakening of the krona in a one-year perspective. The two implied volatility measures have been linearly weighted, i.e. without taking into account that the correlation between USD/SEK and EUR/SEK may be less than one.

Inflation uncertainty and correlation are also approximated with this method, using historical data. Year-on-year changes during the period December 2001 to December 2002 are used for both parameters.

6. References

"Central Government Debt Management – Proposed Guidelines", document number 2003/1665, Swedish National Debt Office.

"Duration, Maturity Profile and the Risk of Increased Costs for Central Government Debt, document number 2003/1590, Swedish National Debt Office.

RIKSGÄLDS KONTORET
THE SWEDISH NATIONAL DEBT OFFICE

Visiting address: Norrlandsgatan 15 • Postal address: SE-103 74 Stockholm, Sweden
Telephone: +46-8-613 45 00 • Fax: +46-8-21 21 63 • E-mail rgk@rgk.se • Internet: www.rgk.se

Wildeco